Why Algebra?
What Algebra?

Working Group Presentation

Gard Brekke, Mollie McGregor
Jack Abramsky, Michael Bulmer, Hugh Burkhardt, Anne Crawford,
Claude.Gaulin, Per-Eskil Persson, Alla Routinsky

ICMI Algebra Study Conference
University of Melbourne
December 2001

Our focus: the minimum

Algebra for All (most)

in

Compulsory Education

We will NOT discuss:

- **Teaching for 'connections' .....**
  rich activities, not just "EEE"

- **Professional development**
  .... related to the above

- **Teacher knowledge, for the minimum**
  particularly, modelling the real world

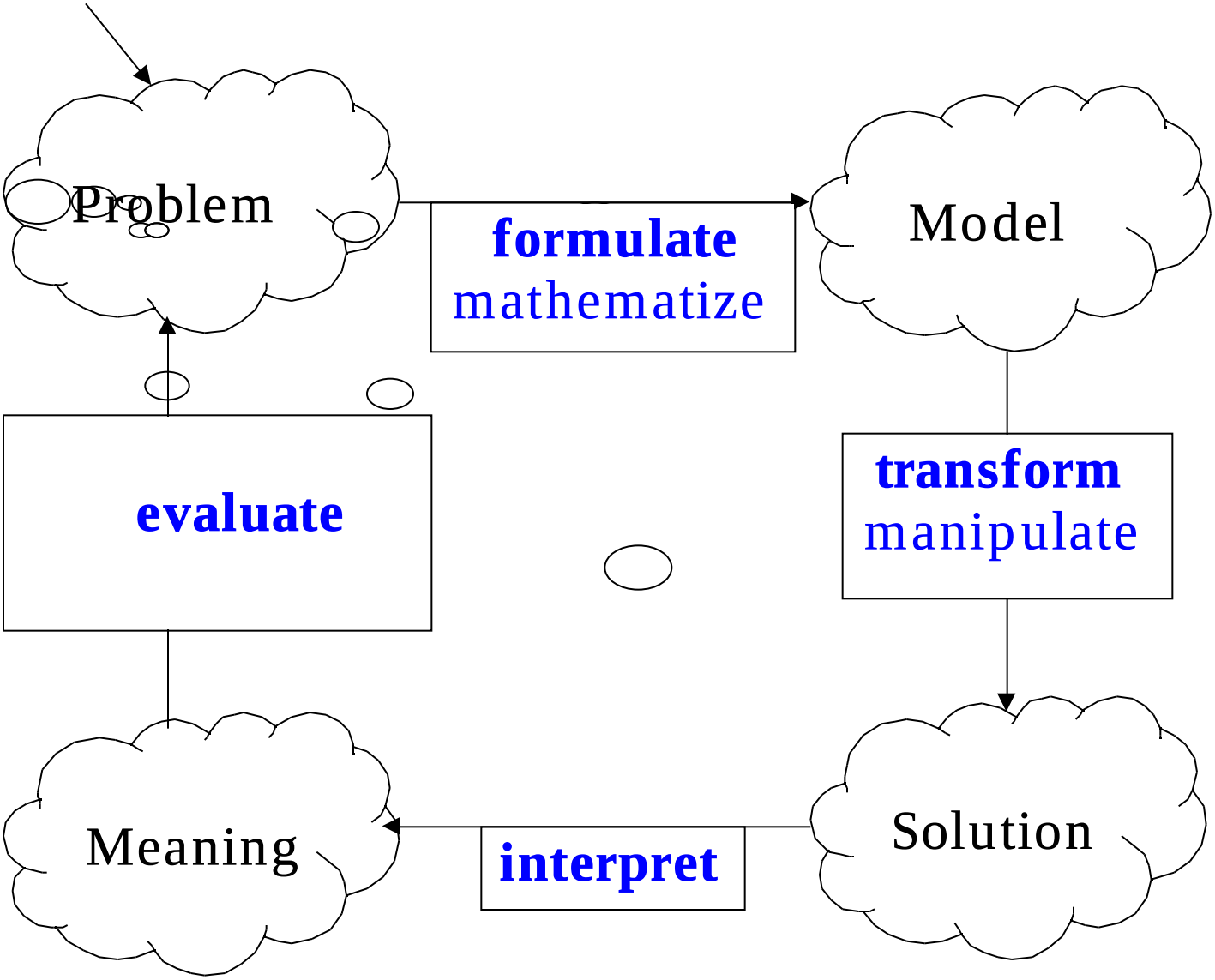
- **Impact of technology on algebra**

- **System change issues**

Why Algebra for All?

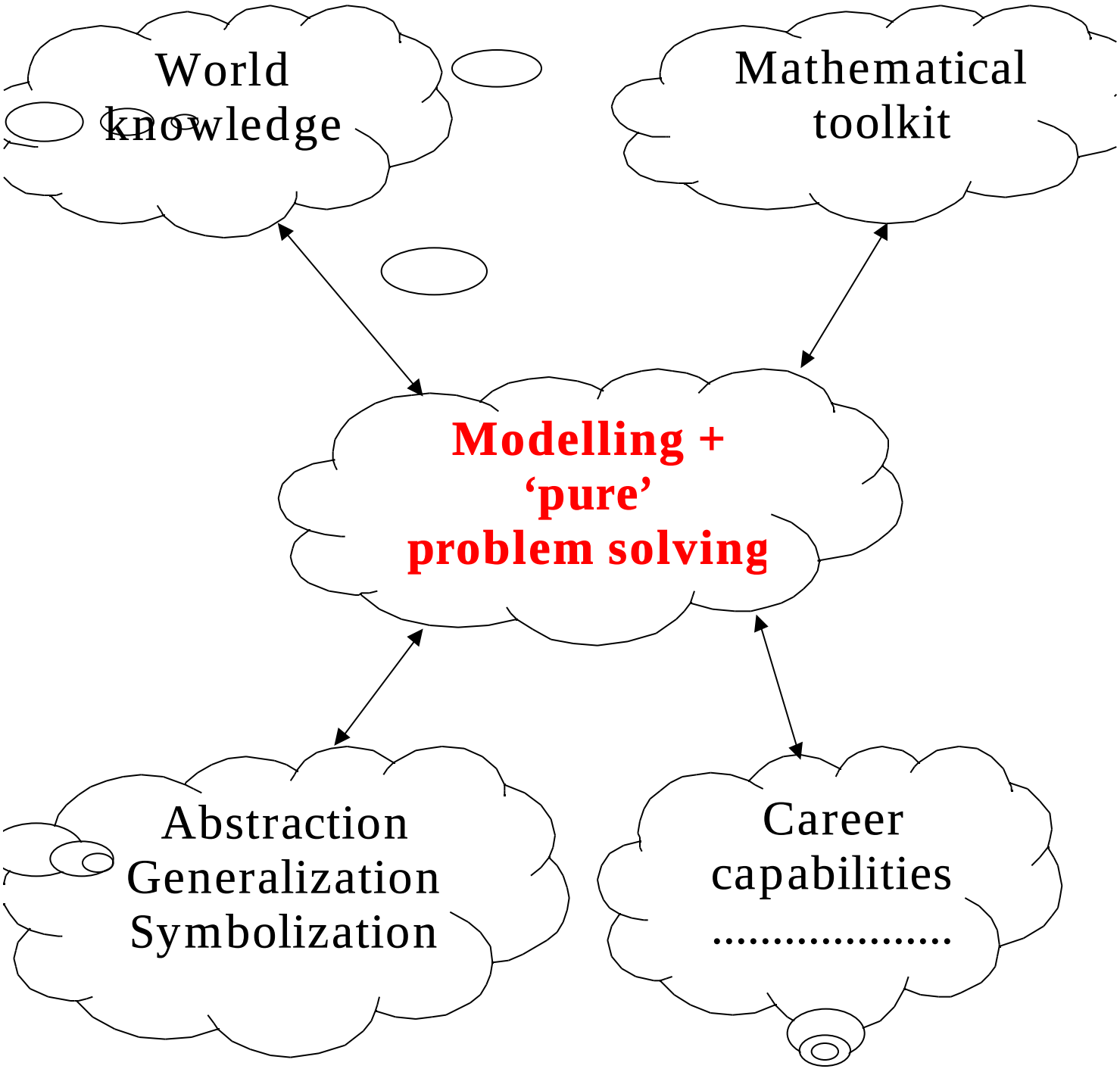
- **societies' increasing demands for maths**
- **technology demands symbolism**
- **equal access to career opportunities**
- **key to higher level mathematics**
- **natural language for problem solving**
- **providing intellectual challenge**
- **supports power of reasoning and critical analysis**
- **"We are abstracting animals"**

Phases of Problem Solving



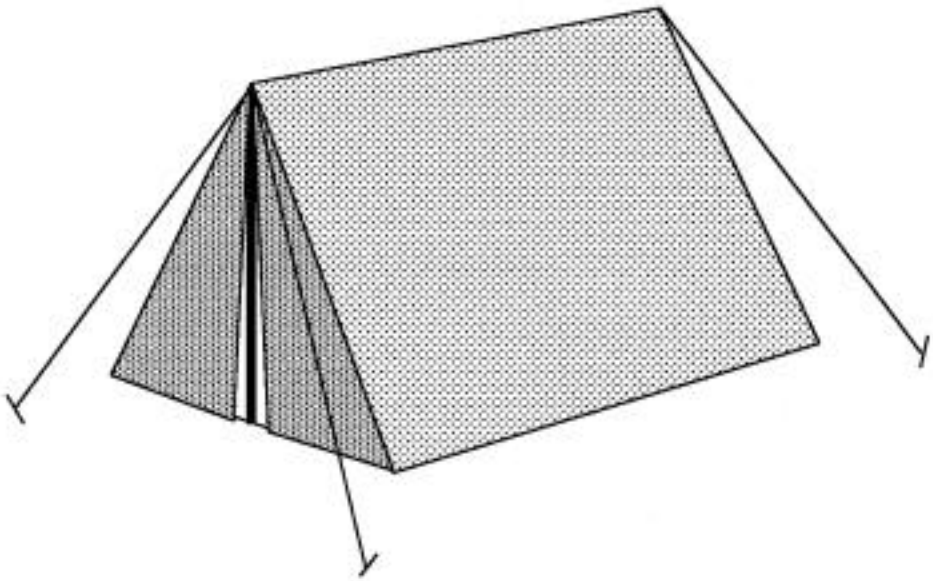
The relative importance of each phase *varies* with the problem (type)

A Designing Structure



Examples

Design a Tent



- It must be big enough for two adults.
- It must be big enough to move around in while kneeling down.
- The bottom of the tent will be made from a thick rectangle of plastic.
- The sloping sides and the two ends will be made from a single, large sheet of canvas.

Supermarket Carts

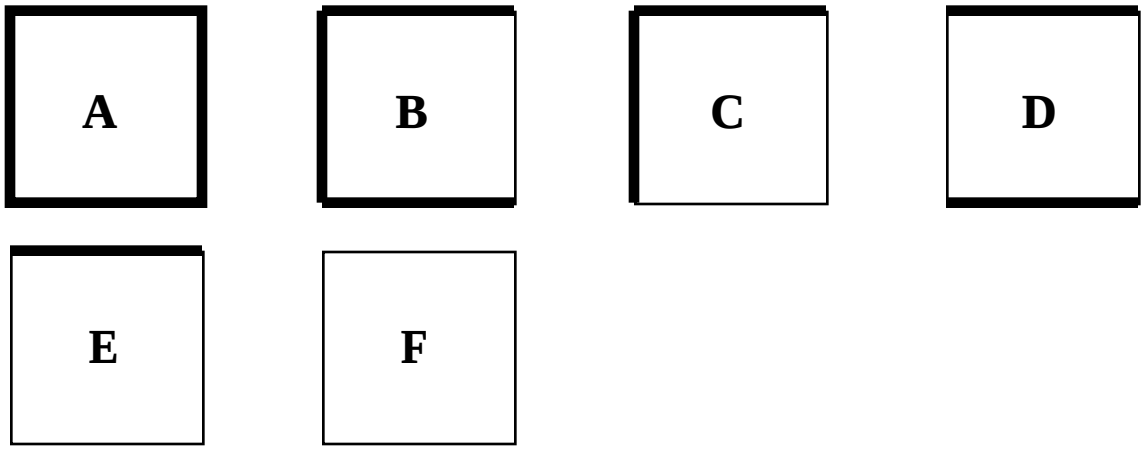
The diagram below shows a supermarket cart.
It also shows 12 supermarket carts "nested" together.
The drawings are $\frac{1}{24}$ th **real size.**

1. Create a rule that tells you the length of storage space (S) needed when all you know is the number of supermarket carts to be stored.
2. Show how you can figure out the number of carts that can fit in a space S meters long.

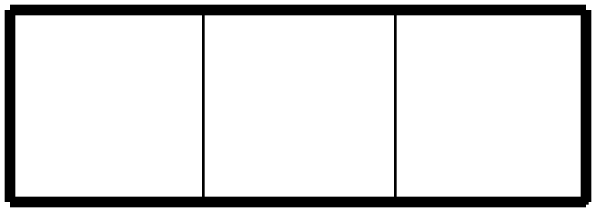
Border Tiles

When tiling, people sometimes use special tiles that have borders on them.

The six possible tiles are shown below.



Tiles are placed together to make rectangles with dark borders around the perimeter. eg



Write a general rule for the number of each type of tile in a class of rectangles (eg 2 x n).

Justify or explain why your rules work for all the rectangles in the class.

Do this for as many classes of rectangles as you can.

Fibonacci Sequences

1, 1, 2, 3, 5, 8, 13, 21, 34, 55, 89, 144, 233, ...

Fibonacci puzzles

Fill in the missing number in these Fibonacci sequences:

1	3	4	7	11	

3	1				14

7			15		41

The next three are more difficult!

3					49

5					50

8					9

Try to find a good method for solving puzzles like these difficult ones. Use algebra if it helps.

Explain your method, and make up some examples to show how your method works.

Aspects of
Algebraic Thinking

- **abstraction and generalization**
- **a sense of variables and symbols**
- **symbolic ⇔ other representations**

- **strategies for formulating symbolic models, using your mathematical toolkit**

- **tactical solution planning**
  including anticipation, route ....
- **tactics for manipulating, eg**
  doing and undoing
  balancing .....
- **avoiding unnecessary calculation**

- **interpreting and evaluating solutions**
- **explaining and justifying**

Examples

SKETCHING GRAPHS FROM WORDS

Hoisting the flag

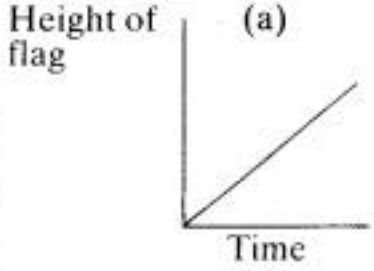
Every morning, on the summer camp, the youngest boy scout has to hoist a flag to the top of the flagpole.

(i) Explain in words what each of the graphs below would mean.

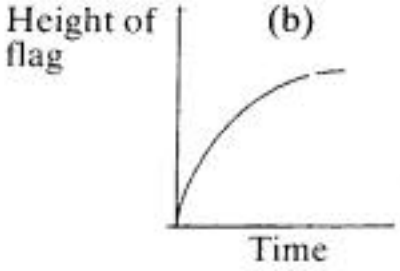
(ii) Which graph shows this situation most realistically? Explain.

(iii) Which graph is the least realistic? Explain.

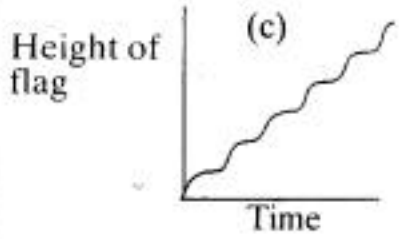
(a)



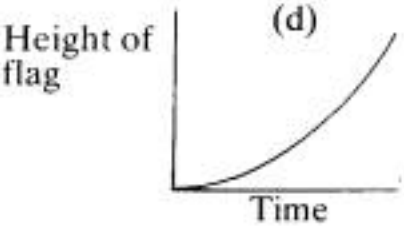
(b)



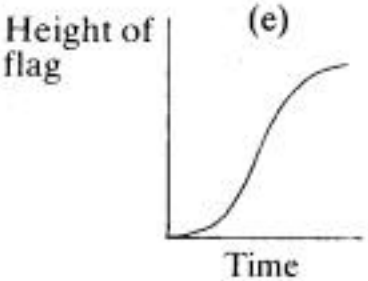
(c)



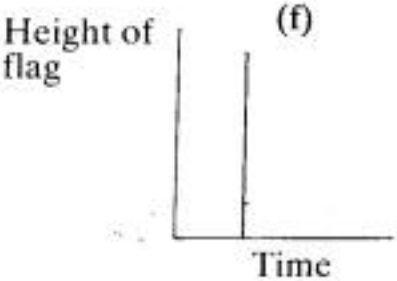
(d)



(e)



(f)



Always, Sometimes or Never ?

This page is for you to discuss with a partner.
Make notes on anything you learn.

Decide whether each of the following statements is

Always True (it is true for *all possible* values)

Sometimes True (it is true for *just some* values)

or **Never true** (there are *no* values which make the statement true)

If you think that a statement is sometimes true, then say which numbers make it true.
If you think that a statement is never true, then say how you can be sure.

1	$n + 5 = 11$	2	$q + 2 = q + 16$
3	$2n + 3 = 3 + 2n$	4	$2t - 3 = 3 - 2t$
5	$3 + 2y = 5y$	6	$p + 12 = s + 12$
7	$4p$ is greater than $9+p$	8	$n+5$ is less than 20
9	$2(x + 3) = 2x + 3$	10	$2(3 + s) = 6 + 2s$
11	x^2 is greater than 4	12	$x^2 = 5x$
13	$2x^2 = (2x)^2$	14	$9x^2 = (3x)^2$

Note: Statements which are always true are called Identities.

Backward Sums

Write down a 2-digit number.
Write it down with the digits reversed.
Add the two numbers together.

Explain
why the sum is always divisible by 11?

Minimal Skills

Know how to:

- **substitute in formulas:**
  proportional, linear, reciprocal, exponential, quadratic
  + (if calculator) more complex functions

- **make tables, plot and interpret graphs of functions,** noting eg max , min, zeros, gradient..

- **interpret and sketch graphs of situations**
  noting eg max , min, zeros, gradient....

- **solve f(x)=0, and f(x)=g(x) graphically**
  solve f(x)>g(x) or f(x)<g(x) graphically

- **solve linear equation:** 1 eqn, 1 variable

- **translate between representations**

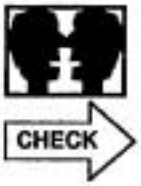
Use the above in formulating models appropriately with numbers, graphs and functions

Examples

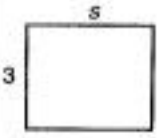
Making and Simplifying Expressions

This piece of work contains common mistakes.
The atudent was asked to find expressions for perimeters, areas and volumes and write them as simply as possible.

Discuss the answers with your neighbour and correct the mistakes you find.

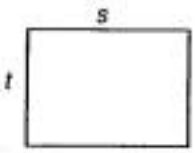


1.



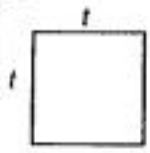
Perimeter = 3+s+3+s
= 6ss
Area = 3xs = 3s

2.



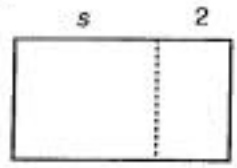
Perimeter = s+t+s+t
= 2s2t
Area = sxt = st

3.



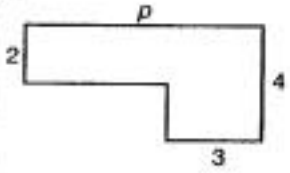
Perimeter = txt = tt
Area = t+t+t+t = 4t

4.



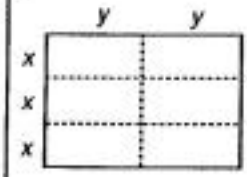
Perimeter
= s+2+t+s+2+t
= 22sstt = 4+2st
Area = txs+2

5.



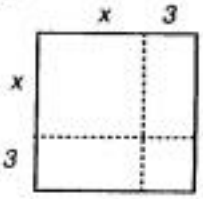
Perimeter
Don't know the sides
Area = 2p+12

6.



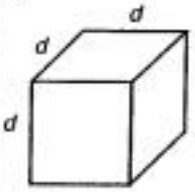
Perimeter = 3x+2yx2
Area = 3x x 2y

7.



Perimeter
= 2(x+3+x+3)
= 2(2x+6)
Area
= xx+3x+3x+9
= 8x+9

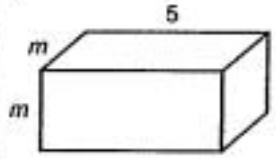
8.



Volume = dxdxd = 3d

Surface Area
= dxdx6 = 2dx6 = 12d

9.



Volume = mm5

Surface Area
= mm+mm+5mx4

Specialist Growth

In 1998 there were 429,316 people employed in the US as computer support specialists.

Accordingly to the Bureau of Labor Statistics, that number is expected to grow to 868,674 by 2008.

Assuming a constant annual increase in the number of specialists, what will the number be in 2006.

Give the algebraic model for this growth.

Translation Skills

From / To	**Situations Words**	**Tables**	**Graphs**	**Formulae, symbols**
Situations Words		interpret	***interpret graphs***	recognizing formulae
Tables	collect data		**read off points**	***calculate values***
Graphs	***sketch graphs***	**plot graphs**		**plot graphs of functions**
Formulae, symbols	***model situations***	‘ algebraic fit’ or model data	‘ graph fit’ or model graph	**manipulate**

Bold: Traditional school emphasis
Italic: Increased emphasis recently

Task Types

Task Type will be one of:

- open investigation
- non-routine problem
- design or plan
- evaluation and recommendation
- review and critique
- re-presentation of information
- definition of concepts
- technical exercise

Aspects of understanding, or mathematical power

Imitation

Retention

Explanation

Adaptation

Extension