Panel 3

The first report from the OECD's International Programme for Student Assessment (PISA) was released recently, see http://www1.oecd.org/publications/e-book/9601141e.pdf, or a short version http://www.pisa.oecd.org/knowledge/summary/intro.htm.

Two major and two minor aspects are used to organise the mathematical literacy. The major aspects are described as *mathematical competencies* and *mathematical big ideas.* Eight competencies are described, of which three are clearly related to algebra. These are:

Modelling skills

This includes:

- Structuring the field or situation to be modelled
- Mathematising and "de-mathematising"
- Working with a mathematical model
- Validating the model
- Reflecting, analysing and offering a critique of a model and its results
- Communicating about the model and its results

Representation skills

This includes:

- Decoding
- Interpreting and distinguishing between different forms of representation of mathematical objects and situations and the interrelationships between the various representations
- Choosing, and switching between, different forms of representation, according to situation and purpose

Symbolic, formal and technical skills

This includes:

- Decoding and interpreting symbolic and formal language and understanding its relationship to natural language
- Translating from natural language to symbolic/formal language
- Handling statements and expressions containing symbols formulae
- Using variables, solving equations and undertaking calculations

What kind of algebraic knowledge is needed to meet the three competencies above?

Three competency classes are defined to operationalise mathematical competencies. These are:
Class 1: reproduction, definitions, and computations,
Class 2: connections and integration for problem solving
Class 3: mathematical thinking, generalisation and insight

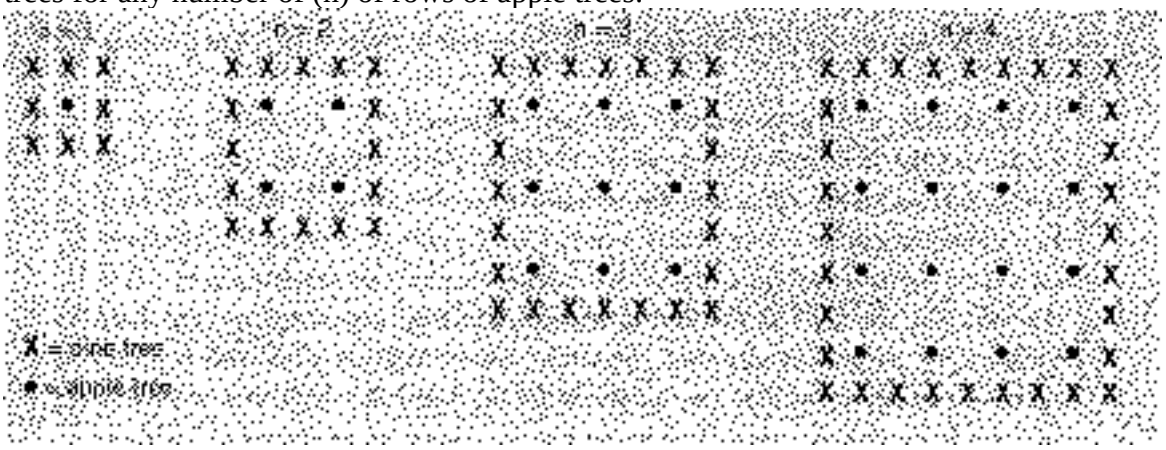
In class 3 students are asked to:

- Mathematise situations and to use mathematics to solve the problem

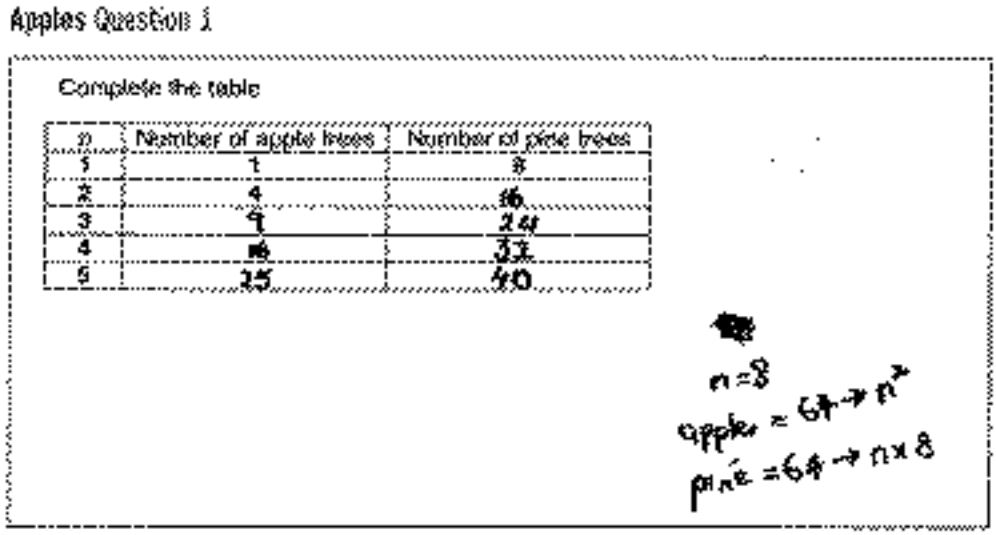
- Analyse
- Interpret
- Develop their own models and strategies and to present mathematical arguments, including proofs and generalisations.

The item *Apples* below is used to illustrate the framework. Examples of Australian students responses are given.

Scenario
A farmer plants apple trees in a square pattern. In order to protect the threes against the wind he plants pine trees all around the orchard.
Here you see a diagram of this solution where you can see the pattern of apple trees and pine trees for any number of (n) of rows of apple trees:



The first question asks students to extrapolate from the diagrams given and complete a table to show how the number apple trees conifers increase as the size of the orchard is increased. An example of a correct response is:



This item belongs to Competency Class 2.

The second item provides two algebraic expressions to describe growth in numbers of the two kinds of threes as the number of rows increases. Also this item belongs to Competency Class 2. One example of response is shown below.

Apples Question 2

There are two formulae you can use to calculate the number of apple trees and the number of pine trees for the pattern described above:

Number of apple trees = n^2

Number of pine trees = $8n$

where n is the number of rows of apple trees.

There is a value of n for which the number of apple trees equals the number of pine trees. Find the value of n and show your method of calculating this.

value of n is 8. Worked out because 8 x 8 is the same as 8^2.

Question 3 is a Class 3 item

Apples Question 3

These responses earned full credit of two points:

Suppose the farmer wants to make a much larger orchard with many rows of trees. As the farmer makes the orchard bigger, which will increase more quickly: the number of apple trees or the number of pine trees? Explain how you found your answer.

The number of apple trees will increase more quickly because that number, say 30, will be squared and so increase quicker than 8 x 30. The next sequence could be 31, and this time itself will give an ever increasing no. than something always multiplied by 8 which will obviously only go up 8 each time

The number of apple trees would increase faster because its square (n^2) not just multiplied ($8n$).