This paper describes how a researcher developed task and four different data collection instruments provide evidence for the identification of various affordances of a technology-rich teaching and learning environment (TRTLE) that were perceived and/or enacted by Year 9 students in their solution of a linear function task utilising a graphing calculator. Each instrument proved valuable in the identification process, with post task interviews being particularly useful in identifying rejected affordances and others that had not been perceived until post task questioning and reflection.

INTRODUCTION
At ICME 5, Pollak (1986) insightfully commented on the power of electronic technologies to transform mathematics, mathematical activity, and mathematics thinking which many proponents of the use of these technologies in secondary school classrooms would have expected to be realised by now. Despite several decades where electronic technologies have played a role in many industrialised countries in the mathematics classroom and curriculum documents have advocated their use, the transformational power of a technology-rich teaching and learning environment (TRTLE) in secondary classrooms has yet to be universally realised. This transformation includes teaching old things better, new mathematics that was unable to be taught without technology, and increasing the breadth and depth of key concepts such as function (DeMarios & Tall, 1996). Many affordances that would be useful in the teaching and learning of function are offered by classroom environments involving these technologies. However, the realisation of any affordances depends not only on “the technological tool, but [also] on the exploitation of these affordances embedded in the educational context and managed by the teacher” (Drijvers, 2003, p. 78).

The term affordance was first coined in 1966 by the perceptual psychologist J. J. Gibson who used the word for

something that refers to both the environment and the animal in a way that no existing term does. It implies the complementarity of the animal and the environment … . They are not just abstract physical properties. (1979, p. 127)

Gibson (1977) considered affordances to be relationships between objects and actors involved in interactive activity. They are what the environment offers to a particular animal. Following Scarantino (2003), the affordances of a teaching and learning environment incorporating electronic technologies will be taken to mean the offerings of such an environment for both facilitating learning (the promises or
positive affordances) and impeding learning (the threats or negative affordances). An affordance of a TRTLE is the opportunity for interactivity between the user and the technology for some specific purpose, for example, check-ability.

“Affordances describe how the interaction between perceiver and perceived works - and that is exactly what we need to understand in educational research” (Laurillard, Stratfold, Luckin, Plowman, & Taylor, 2000, p. 3). “Research examining the concept of affordances is crucial if we are to build … a more flexible design orientation to the practice of education” (Pea, 1993, p. 52). Electronic technologies “enhance the individual’s capacity to act” (Smitsman & Bonger, 2003, p. 176) but “cannot be considered independently from … the environment” (p. 173).

THE STUDY

The data analysed here are part of a larger study which aims to construct a theory (Strauss & Corbin, 1990) of how teachers and secondary students perceive and enact affordances in a TRTLE so as to maximise the learning of functions. From a research perspective it is necessary to establish what conditions exist in TRTLEs enabling or impeding the realisation of offered affordances for the teaching and learning of functions. The purpose of this paper is to determine which combination of research instruments provide optimal evidence for identifying affordances of a TRTLE perceived by Year 9 students in their solution of a linear function task utilising a graphing calculator.

METHODOLOGY

A case based approach has been adopted for the study proper. In intrinsic case studies “the case itself is of primary, not secondary, interest” (Stake, 1995, p. 171) whereas in instrumental case studies “the case study serves to help us understand the phenomena or relationships within it” (p. 171) as is the situation in this study. The phenomenon being studied is the perception of affordances by secondary teachers and students after the teaching of functions in a TRTLE. The study is an instrumental multiple case study, but only data from one teacher’s classes will be analysed in this paper.

A case based approach typically begins with a descriptive phase within which exploratory-descriptive work is undertaken the goal of which is to examine, investigate, and document the phenomenon on its own terms in an open-minded fashion (Edwards, 1998). In the aspects of the case to be reported here, the purpose of data collection has been to understand the affordances perceived by the Year 9 students in a TRTLE and to evaluate the research instruments being used for this purpose.

Forty two students (14-15 year olds), from two Year 9 classes taught by the same teacher (24 from class A and 18 from class B), participated in the data collection. Various electronic technologies, including both laptop computers and graphing calculators owned by the students, have been used extensively by them throughout
Year 9. Both classes had completed a unit of work on linear functions utilising a range of electronic technologies before data collection began.

A graphing calculator program, *Hidden*, developed by the author, was utilised as the basis of the *Hidden Function* task to generate a (linear) function and edit TABLE settings controlling the display of TABLE values by changing from automatic to user initiated display of both. On execution of the program, students were presented with a blank TABLE. The task was attempted in pairs. The pairs were asked to identify several ‘hidden functions’ and then describe a general method to do so. The program and the methods for accessing numerical data in the TABLE were demonstrated by the author at the beginning of the task. A second task version provided a split-screen view with TABLE and GRAPH side-by-side when the TABLE was accessed.

The task was implemented once in each class during a 50 minute lesson. In both classes two focus pairs were video taped in their regular classroom setting, during the task solution. Six and four pairs of students, from class A and B respectively were interviewed immediately after the task sessions. These included all focus pairs. In Class B the split-screen version of the task was used with four pairs. An application from Freudenthal Institute website that records key strokes of graphing calculator users, *KeyRecorder*, was successfully implemented with five pairs of students in Class B. The application was not functional when class A was administered the task.

Data collected included task record sheets (21 pairs), video transcripts (4 focus pairs), key screens (5 pairs), and post task interviews (10 pairs). The responses of the student pairs to the *Hidden Function* task and subsequent interviews of some pairs were used to document the range of affordances perceived by the students either during the solution process or subsequent discussion of the task in the interview.

**AFFORDANCES PERCEIVED BY THE STUDENT PAIRS**

All students successfully interacted with the *PROGRAM: Hidden* and the graphing calculator TABLE feature to generate numerical data from which to engage with the task. Eleven additional affordances were perceived by some pairs either during their task solution or the post task interview. These were affordances of the TRTLE to:

1: Access the general equation of the Hidden Function (i.e., $y_0 = mx+c$) in the TABLE heading and recognise the function as linear and/or the need to identify $m$ and $c$.

2: Use the HOMESCREEN to undertake calculations (e.g., to determine required values, or evaluate values of a conjectured function).

3: Generate numerical values in the TABLE and link these to the graphical representation of the function, (e.g., to determine $y(0)$ to find the $y$ intercept).

4: Use the LISTS and LinReg (linear regression feature) to determine the algebraic representation of a function.

5: Make deliberate choices of entering consecutive values for $x$ in the TABLE to simplify the solution path.
6: Use the graphical representation to identify the hidden function as linear (i.e., visually or by displaying $y_0 = mx+c$ in this representation).

7: To access stored values in the graphing calculator (e.g., ALPHA M, ALPHA C) to identify the gradient and y intercept of the function.

8: Use the graphical representation to identify the sign of the gradient, or the y intercept, or to think about the function in some other way.

9: Use the TABLE to perform a local check of conjectured values of the function (e.g., by generating additional pairs of numerical values).

10: Use the function window ($y =$) and TABLE to provide a numerical representation of a given function to verify a conjectured function as the hidden function.

11: Use LIST OPERATIONS to verify a conjectured function (i.e., with data in two lists enter a conjectured function rule in a third list and compare values).

The frequency of identification of these affordances and the sources of evidence for this identification are shown in Table 1. Sources of evidence are record sheets (RS), video transcripts (focus pairs only) (VT), keys screens (KS), or post task interviews (I). Evidence for a pair perceiving an affordance may be gained from multiple sources. Affordances that student pairs considered only during the post task interview are indicated by an asterisk (*).

Of the eleven affordances listed in Table 1 the record sheet was evidence for 7 of these, the video transcript for 7, the key screen record for 5, and post task interview for 10. Whilst the greatest number of instances was identified using record sheets (21

<table>
<thead>
<tr>
<th>Affordance Number</th>
<th>Number of pairs class A, class B</th>
<th>Source of Evidence (number of pairs)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>RS</td>
</tr>
<tr>
<td>1</td>
<td>1, 4</td>
<td>2</td>
</tr>
<tr>
<td>2</td>
<td>2, 4</td>
<td>0</td>
</tr>
<tr>
<td>3</td>
<td>7, 1</td>
<td>6</td>
</tr>
<tr>
<td>4</td>
<td>2, 1*</td>
<td>2</td>
</tr>
<tr>
<td>5</td>
<td>4, 1</td>
<td>1</td>
</tr>
<tr>
<td>6</td>
<td>0, 2</td>
<td>1</td>
</tr>
<tr>
<td>7</td>
<td>0, 1*</td>
<td>0</td>
</tr>
<tr>
<td>8</td>
<td>6, 5</td>
<td>1</td>
</tr>
<tr>
<td>9</td>
<td>4, 7</td>
<td>8</td>
</tr>
<tr>
<td>10</td>
<td>0, 1*</td>
<td>0</td>
</tr>
<tr>
<td>11</td>
<td>2*, 1*</td>
<td>0</td>
</tr>
</tbody>
</table>

Table 1: Frequency and source of perception of various affordances.
in total), this instrument was used to collect data from all pairs unlike the other instruments. Three affordances, 7, 10, and 11, the latter two being opportunities for solution verification, were identified by only the post task interview. A further three affordances, 1, 8, and 9, were identified by all data sources. One of these will be used to illustrate how each instrument contributed evidence for perception of affordances.

**EVIDENCE FOR AN AFFORDANCE**

One affordance of the TRTLE was to use the graphical representation to identify the sign of the gradient, or the $y$ intercept, or to think about the function in some other way (Affordance 8). The perception of this affordance was identified from the record sheet of Hala and Kay, where in describing their general method they stated, “Also, look at the graph to get an idea of what operations and how large they are”. Additional evidence from their post task interview suggested the graph allowed them to identify the sign of the gradient. Whilst other pairs may have perceived this affordance, no other record sheets provided evidence for this. The video transcript of Michelle and Tim provided evidence of Tim using the graphical representation to think about the relationship between the numbers.

Tim: Timesing by 10.1, so 30, the $y$ value, is $y$ the across, is it? [Whilst looking at the graphical representation on the split screen view, he runs his finger across screen above and parallel to the $x$ axis].

Michelle: The $x$ axis is along. The $y$ axis is down.

Tim: That's right. So it's hitting the $y$ [indicating by running his hand vertically across the screen parallel to the $x$ axis], $x$ axis at 30 and the $y$ axis at 310.

The key screens of Di and Fiona provided evidence of their observing the graphical representation on three occasions as they unsuccessfully tried to identify a third ‘hidden function’. Their view of the function in the standard window did little to assist this identification, however. In contrast Obi and Luke’s key screens suggest the graphical representation afforded them the opportunity to think about the ‘hidden function’. This is evidenced by the time spent viewing the graph and moving the cursor around the screen. In conjunction with their video transcript, it is apparent that not only did the graphical representation allow this thinking to occur but also it facilitated enactment of the more specific affordance offered by the graphical representation (Affordance 6) as they identified the general equation and $y$ intercept.

Obi: [Presses TRACE] $y = mx + c$ according to this.

Luke: I thought it was $y = mx + c$.

Obi: Just write that down. [Passes graphing calculator back to Luke]

Luke: Yep, say, yes ... Look at this [shows screen to Obi]. Zero is three. $Y = 3$.

Post task interviews provided evidence of extra affordances perceived but rejected during task solution. The conversation between interviewer and students allowed reflection on the task by the pairs and consideration of affordances other than those utilised or rejected during task solution. The instrument, therefore, allowed identification of additional affordances that students could have used. These
affordances may well have facilitated solution verification had the students felt there was a need for this. One pair, Rick and Kris, rejected the affordances offered by the graphical representation during their task solution although this was not explicitly evident from their record sheet or the video transcript. The following dialogue shows Kris accessed the graphical representation, and considered the offered affordances. Rick, focussed on an independent solution pathway at the time, was unaware of this.

Interviewer: Could you have used the graph? Could you have solved it graphically?
Rick: No, no graph.
Kris: We did look. We could find out the y intercept using it and then ...
Interviewer: So you did look at the graph in the class then?
Rick: Never!
Kris: Once. We did once but then …

However, the perceived affordance was rejected for other affordances, both students clearly believed other offerings of the TRTLE enabled more efficient task solution.

Kris: Although you can use a, umm, function, the intercept function to find the y intercept, but it is easier to just go to table and to do zero.
Rick: Zero.

Interviewer: But you could have done it with the graph as well?
Kris & Rick: Yes

Tim and Michelle used a split-screen version of the task. Whilst TABLE values needed to be deliberately selected, a portion of the graph was visible at all times in the graph window. Whilst saying they ignored the affordances offered (contrary to video tape evidence cited earlier), this pair actually perceived and rejected these as not meeting their current needs.

Interviewer: Are there other ways that you could have found the hidden function?
Michelle: There is usually a graph we could probably have used, which ...
Interviewer: Didn't you have a graph on your screen?
Tim: Yeah.
Michelle: Yeah, but we didn't look at it.
Tim: It didn't have any really [pause] numbers. It just had [pause] the line.

Interview questions asking student pairs not only about the choices they made, but also about other choices that could have been made during their task solution enabled the gathering of evidence of a broad range of perceived affordances of the TRTLE.

THE VALUE OF THE POST TASK INTERVIEWS

Of particular interest was the enactment of affordances for checking or verification of the solution. Checking, where it occurred, was generally at a local level. The post task interviews and in particular questions about other choices and how their equation could be checked allowed the identification of affordances that students may not have
considered during task solution, but were possibilities in an environment where solution verification was expected. Evidence of this is presented for Obi and Luke. After spending much of the allocated task time engaged in off-task behaviour, Obi and Luke’s record sheet showed numerical data generated for five hidden functions with only two algebraic representations recorded (both correctly). The post task interview began in a similar fashion as they generated five ordered pairs for a ‘hidden function’ then appeared to have little idea how to proceed. They started to ‘wander around’ the various calculator features, reflecting their behaviour at times during task solution. The graphical representation was considered and rejected, as they had forgotten how it proved helpful previously. They continued searching calculator features until Obi appeared to have an idea whilst looking at a statistics menu.

Obi: Do you want to do that one? ax + b =, umm.
Interviewer: You went to linear regression? Have you used linear regression in class?
Obi: No, we haven't.

Their teacher who expected most students would use linear regression on the calculator to solve the task would disagree with this comment. This pair recognised an error message meant they had no data in their LISTS, proceeded to enter their data into the LISTS, and used linear regression to correctly identify the ‘hidden function’ (Affordance 4), supporting the teacher’s comments from a post task interview.

In response to questions about checking their equation, Obi immediately suggested using LIST OPERATIONS but then said he preferred to use the function window (y=). Obi worked with Luke to use LISTS to verify their function (Affordance 11).

Interviewer: Okay. Then L3 = 4*L1+ 6, yes? [Describing what the pair entered.] And?
Luke: And the answer is 26, 42, 54, 66.
Interviewer: So LIST 2 and LIST 3 are the same?
Obi: Yeah.

Obi was then asked about his alternative idea to check their solution. The pair entered their function as y1, accessed the table values for y1 and noted these were identical to the values of the hidden function (Affordance 10). This pair, although relatively unsuccessful during task solution, clearly, when focussed on the task were able to successfully perceive and enact a number of affordances of the TRTLE that allowed both task solution and solution verification.

**DISCUSSION OF RESULTS**

This study has shown that in the TRTLE’s involving these two classes a range of affordances were perceived and rejected or enacted for various purposes. Each of the four instruments proved valuable in identifying these affordances. This identification occurred through use of a single instrument, multiple methods of identifying the same instance of an affordance perceived, and, at times, two or more instruments contributing evidence which when combined identified the perception of an
affordance, that evidence from a single source was not enough to substantiate the perception. The post task interview, in particular, provided evidence of affordances that the other instruments did not. Questions relating to alternative choices available to the student pairs with respect to particular features of the graphing calculator, or for the solution pathway, and ways in which they could check or verify their solution proved most informative in this regard. Whilst the use of the post task interview appears to provide the greatest evidence of perceived affordances, each instrument brought valuable information to light. This was sufficient to warrant further use of each in future data collection for this study¹.

References


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