MEDIATING THE COGNITIVE DEMAND OF LESSONS IN REAL-WORLD SETTINGS

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A challenge for secondary mathematics teachers is to be able to design learning experiences so as to manage the level of cognitive demand of lessons and tasks to ensure students will engage with these and learn from the experience. A framework for engineering the cognitive demand of tasks, lessons, and lesson sequences will be illustrated within the context of Year 9/10 using a teacher designed task implemented during the year.

Introduction

According to Evans (1991),

> One way of characterising teaching tasks is through the kind of *cognitive demand* [italics added] they impose on the learner: whether these consist in the requirement of specific procedures elicited by particular cues, recall of specific knowledge, development and application of structured conceptual knowledge, or higher order procedures involving interpretation, transfer of rules to unfamiliar materials, or the combination and modification of procedures. (pp. 125-126)

In the RITEMATHS project (HREF1) involving 6 secondary schools, the Universities of Melbourne and Ballarat, and Texas Instruments, one aim is to develop guidelines for managing increased cognitive demand of lessons where task contexts involve real-world applications in a technology rich learning environment. From a teaching perspective the management of the cognitive demand of teaching tasks in such an environment can be mediated through (a) task scaffolding, (b) task complexity and (c) complexity of technology use.

Mediators of Cognitive Demand: Task Scaffolding

Task scaffolding can be described as "the degree of cognitive processing support that the task provides the task solvers, enabling them to solve complex tasks that would be beyond their capability if they depended on their own cognitive resources" (Stillman, 2001, p. 103). The degree of support a teacher may want to provide for a task is related to its purpose and placement in a lesson sequence. Task scaffolding comes through the structuring of the task, the type of technology chosen, and whether technological assistance rather than by-hand calculation, is privileged by the task. It also comes from *whose choice* it is (task setter's or task solver's) to make decisions about all of these when solving the task.

In tasks with *high* task scaffolding, the task setter structures the solution pathway by the way the task is posed. This usually results in a reduction in the number of decision points the task solvers have to face and resolve for themselves. Tasks with *low* scaffolding allow for student generated solutions where students make their own assumptions, construct their own model of the situation and make event-driven decisions throughout the solution attempt. When using tasks set in real-world contexts, these two extremes of the task scaffolding continuum result in a continuum of observed modelling outcomes ranging from *supplantive modelling* where the modelling structure is supplied by the task setter to *generative modelling* where the students generate the modelling themselves (Figure 1). This suggests that if we believe student generated solutions are important in the learning process then we need to be prepared to provide tasks low in scaffolding.

Continuum of Task Scaffolding

High ⟷ Low

Continuum of Observed Modelling for Real World Tasks

Supplantive Modelling ⟷ Generative Modelling

Figure 1. Relationship between continua for task scaffolding and modelling outcomes for real world tasks.

Ideally, over the years of secondary schooling students should be able to progress along the continuum from supplantive modelling towards generative modelling by being engaged in lessons where teaching, learning, and assessment tasks of accessible, but increasing, levels of cognitive demand have been provided. Within a year level, and even within a unit, teachers may endeavour to ensure there is some progress by regulating the level of cognitive demand in lessons and in tasks used in lessons through a commensurate reduction in the level of task scaffolding.

Mediators of Cognitive Demand: Task Complexity

The complexity of a real world task can be shown by identifying and assessing the level of those attributes of the task that contribute to its overall complexity. The attributes of a task that contribute to its complexity are potentially numerous (see Figure 2). They contribute to overall complexity via the mathematical, linguistic, intellectual, representational, conceptual, or contextual complexities of the task. These subcategories of task complexity have been refined and augmented by the first author (Stillman, 2002) using a grounded theory approach based on Strauss and Corbin (1990) from categories used by Williams (2002). For each subcategory, properties and their dimensional ranges have been identified. For example, one of the properties of linguistic complexity is orientation of wording which can be mathematical, everyday, or technical. The level of overall task complexity can vary along a continuum from simple to complex. For a particular task, both students and teachers tend to focus on only a subset of attributes which act as indicative cues in assessing overall task complexity. The attributes of a task that contribute to its complexity can be specified by answering relevant questions from each subcategory shown in Figure 2.

Mediators of Cognitive Demand: Complexity of Technology Use

The use of electronic technologies such as graphing and CAS calculators and image digitisers can reduce the cognitive demand of tasks through supplementation and reorganisation of human thought by carrying out routine arithmetic calculations, algebraic manipulations, graph sketching, acting as an external store of interim results, or overlaying visual images within an interactive coordinate system to facilitate analysis. However, the use of these technologies has the potential to bring in a degree of complexity as they transform classroom activity and allow new forms of activity to occur. Regulation of this complexity allows teachers a further opportunity to mediate the cognitive demand of lessons through the careful crafting of tasks for teaching, learning, and assessment. In the RITEMATHS project two types of electronic technologies are being used in real-world settings—analysis tools (e.g., the graphing calculator) and real-world interfaces (e.g., image digitisers such as *GridPic 1.2*, Visser, 2004) which bring a virtual world into the classroom for mathematical exploration. Figure 3 provides a preliminary list of questions that could be considered in relation to technology use to inform decisions the regulation of cognitive demand over time.

TASK COMPLEXITY

General Attributes	*Dimensional Ranges*
LEVEL OF COMPLEXITY	simple...complex
CONCEPTUAL COMPLEXITY	
What is the complexity of the concepts involved?	basic…abstract
Concepts from how many topic areas are involved?	1...many
Where are these concepts in terms of pedagogical development?	early...complete
MATHEMATICAL COMPLEXITY	
How many techniques are involved?	1...many
What degree of rehearsal of required techniques has there been?	cursorily treated...well rehearsed
How obscure is the choice of techniques?	fairly obvious...quite obscure
How complex is each technique?	quite simple...quite complex
How complex is the combination of techniques?	all quite simple...most quite complex
What type(s) of combination of techniques is involved?	conjunction, composition, inverse
How visible are the links between techniques?	fairly apparent...quite obscure
How many steps are involved?	1...many
What is the length of solution?	short...long
How familiar is the problem type?	familiar...unfamiliar
What type of problem is it?	direct taught, reverse taught, direct novel, reverse novel
What type of application is it?	procedural...true application
What amount of mathematical information is given?	Sufficient only...information must be imported
How many mathematical topic areas are involved?	1...many
LINGUISTIC COMPLEXITY	
What amount of guidance is given?	none...high
What is the complexity of vocabulary used?	simple...complex
What is the complexity of sentence structure used?	simple...complex
What amount of information is given in written form?	a little...a lot
How familiar is the wording?	familiar...unfamiliar
What amount of reading is involved?	a little...a lot
What is the orientation of wording used?	mathematical, everyday, technical
What format was used?	point form, 1 paragraph, several paragraphs
What is the relevance of information given?	all relevant, extra information
INTELLECTUAL COMPLEXITY	
Is analysis required?	no, yes
Is synthesis required?	no, yes
How much decision making is necessary?	none...a lot
What amount of thinking is required?	little...a lot
What is the level of challenge of the task?	straightforward...perplexing
How many steps are integrated involving mental co-ordination?	few...many
REPRESENTATIONAL COMPLEXITY	
How many visual representations are given?	1...many
What type of visual representation(s) are given?	none, diagram, graph
Can the task be represented in a diagram/graph?	yes, no
How difficult is it to draw the diagram/graph?	easy...hard
CONTEXTUAL COMPLEXITY	
How familiar is the task context?	familiar...totally unfamiliar
How obscure is the mathematical formulation?	obvious...quite obscure
What type of real-world task is it?	application...modelling task
What level of contextualisation is used?	border...tapestry
What amount of contextual info is there to process & integrate?	a little...a lot
How are assumptions for model formulation specified?	all given...all made by student
What is the nature of the reality of the task context?	contrived...real life

Figure 2. Attributes and dimensions of *Task Complexity.*

COMPLEXITY OF TECHNOLOGY USE	
General Attributes	***Dimensional Ranges***
LEVEL OF COMPLEXITY	simple...complex
Specific Attributes	
How many electronic technologies are involved ?	1...many
How are these technologies used?	analysis tool, real-world interface
How much technological knowledge is required?	little...a lot
How easy is the technology to use?	easy...very difficult
How obscure is the choice of techniques?	fairly obvious...quite obscure
How complex is each technique?	quite simple...quite complex
How complex is the combination of techniques?	all quite simple...most quite complex
How visible are the links between techniques?	fairly apparent...quite obscure
How many steps are involved?	1...many
How many features of the technology are involved?	1...many
What amount of guidance is given?	none...high
How much decision making is necessary?	none...a lot
How many representations can the technology provide?	1...many

Figure 3. Attributes and dimensions of Complexity of Calculator Use.

An Example

The task, *Cunning Running* (see Appendix), will be used to illustrate how these theoretical ideas can be applied in practice. This task addresses modelling of variation using Pythagoras' Theorem and is intended for students in Years 9 or 10. The main task is designed for 2 class periods (100 minutes) plus homework. There is a further fifty minute extension lesson. It is assumed that students know how to calculate the lengths of sides of right angle triangles, given 2 known lengths.

This task involves the investigation of position on a base line of the point where the shortest path occurs. Parameters in the investigation are:

1. Gate positions from the base station line
2. Station positions on the base line.

The investigation requires scale diagrams, numerical and graphical analysis, with the algebraic equation of the model being derived from the by-hand calculation procedures used to calculate the path length.

The Lesson Sequence

In the first lesson, as a precursor to the task, students are taken to the gym where a rope is attached to two points on opposite walls (simulating Gate 1 and Gate 2 in task diagram) and stretched to touch the corner of a base line (Corner A in task diagram). A student takes the rope at Corner A, to see how far she or he can continue to touch the wall with the rope. Is there any slack in the rope? Prior to the procedure being repeated with Corner B, teams of students are asked to mark the position where the rope will reach and where the point will be that allows the greatest slack in the rope. After this practical activity, the task is introduced. Discussion of the meaning of total minimum run length ensues after students have completed their scale plans of a field with 2 gates and 18 check stations. After four by-hand computations of distances for particular stations, the results for the remaining 14 stations are calculated by using the LISTs of a graphing calculator (or a spreadsheet) using the formulae that are the algebraic generalisations of the by-hand calculations.

The second lesson is the investigation of the minimum path length, using the graph of distance from the corner of the field to the stations versus total run length. Class discussion focuses on the minimum length and information contained in the shape of the graph. The equation of the graph is investigated. The algebraic model is constructed from the concatenation of the by-hand steps. The curve for this equation is drawn on the scatter plot.

The activities for the extension lesson involve investigations using dynamic geometry software (e.g., a Cabri applet). In this investigation the positions of the stations and both gates can be considered to vary.

1. What happens to the position of the station for a minimum total run length as Gate 1 distance from A – Gate 2 distance from B approaches zero?
2. If the ratio of Gate1 distance from A: Gate 2 distance from B remains the same, even though the actual distances change, does the position of the minimum station change?
3. At the minimum run length situations compare the lengths
   a. Gate 1 distance from A: Gate 2 distance from B
   b. Hypotenuse 1: Hypotenuse 2
   c. Distance 1: Distance 2
4. Introduction to trigonometric ratios: for the minimum total length positions, compare the ratios Gate 1 distance from A: hypotenuse 1 and Gate 2 distance from B: hypotenuse 2. Other ratios may also be considered.

Mediating Cognitive Demand

The lesson sequence came at the end of a unit on Pythagoras' Theorem in term 1, Year 9, and served as both the culmination of this unit and the lead into the next unit on trigonometrical ratios (lesson 3). The task scaffolding in lessons 1 and 2 is relatively high but not unreasonable for students at this stage of their mathematical development. In particular, tight control is kept over the decision making by the task setter. There is only the choice of technology. In the task description there is little scaffolding of the technical aspects for using the selected technology. For example, if the graphing calculator was chosen as in the version of the task in the appendix, specific features to be used include: LIST of values and Operation on List Values, Graph Display (Plot, Trace, Window), Graph Equation (Y=), and minimum value of the function by use of the TABLE. The omission of written scaffolding is indicative of the level of facility with their calculators that could be assumed for the students in the particular classes. More scaffolding could be provided through demonstration or peer interaction if teachers found this was warranted where students were not at this level of expertise. This would ensure that the cognitive demand of the task was not increased by technology use in such circumstances. Such scaffolding could apply to particular students or the whole class.

The task itself could potentially be quite high in cognitive demand for Year 9 due to overall task complexity. It is a genuine modelling task from a familiar real experience for students at this school. However, the approach taken is towards the supplantive end of the modelling continuum. There are many steps involved but students are provided with guidance in the mathematical formulation of the task, choosing techniques to use, the combination of these techniques, and the representations to use at various stages in the solution. In addition, only sufficient mathematical information is given and all assumptions are stated. The task statement is two pages consiting of both text and a diagram. Potential problems with linguistic complexity are mediated through class discussion of the meanings of terms such as minimum run length and through connections with the visual mental images the students would have from the gym activity.

This task could obviously be used at higher levels of schooling when students have much longer experience with the mathematical concepts and techniques involved but it would then be expected that the approach would be much more towards generative modelling. The mathematical and graphing calculator techniques, for these students, should be well rehearsed and automated thus reducing the cognitive demand coming from task complexity and technology use. So in order to keep the level of challenge appropriate, the explicit scaffolding in the current task statement should be deliberately reduced or totally withdrawn to allow students to generate their own solution pathway where they make their own decisions.

Conclusion

Providing the appropriate cognitive demand for students is not a simple task, be it across a year level or within a particular learning activity. The framework presented here details three opportunities teachers have for mediating the demand of real world tasks in technologically rich learning environments. By carefully orchestrating the interplay between degree of task scaffolding, task complexity, and complexity of technology use, teachers are able to craft lesson sequences involving tasks of appropriate levels of challenge for their students. The example presented here illustrates how a teacher has been able to carefully regulate the cognitive demand of his lessons in order to maximise favourable outcomes for his students. This regulation occurred through careful task and lesson scaffolding to mediate the potential complexity of the task and technology use.

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HREF1
http:// extranet.edfac.unimelb.edu.au/DSME/RITEMATHS/

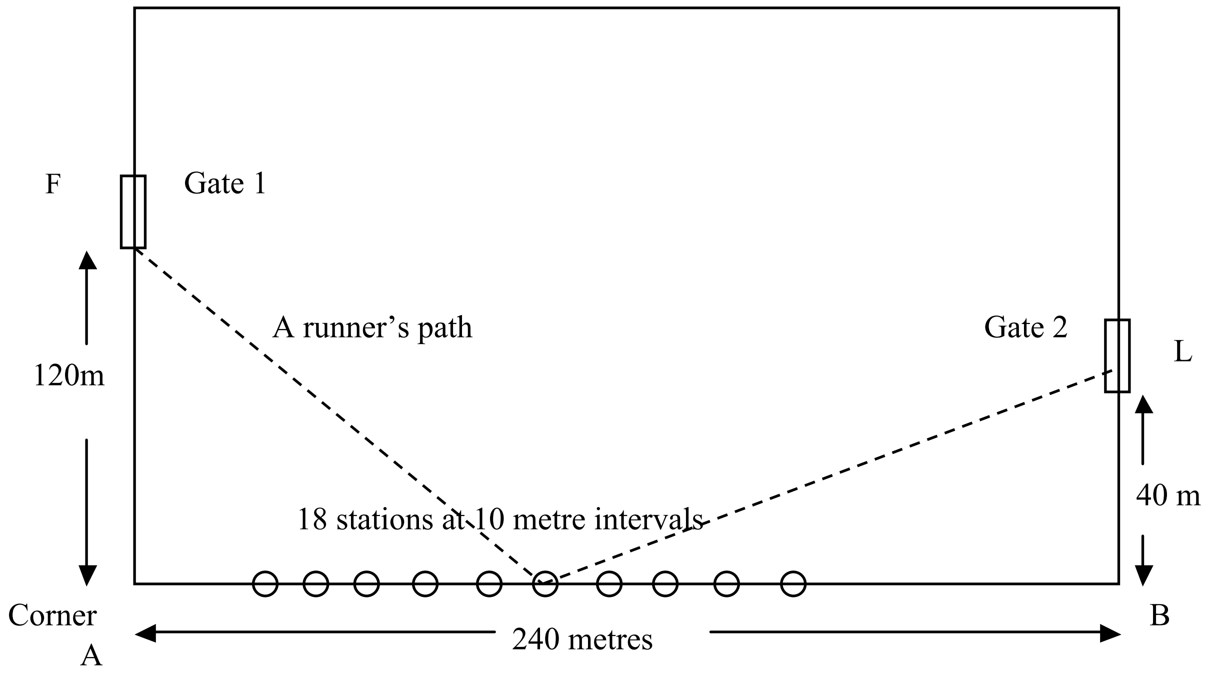
Appendix

ORIENTEERING - Cunning Running

In the Annual "KING OF THE COLLEGE" Orienteering event, competitors are asked to choose a course that will allow them to RUN THE SHORTEST POSSIBLE DISTANCE, while still visiting a prescribed number of CHECK POINT STATIONS.

In one stage of the race, the runners enter the top gate of a field, F, and leave by the bottom gate, L. During the race across the field**,** *they must go to one of the stations* on the bottom fence. There is one station per competitor. Runners claim a station by reaching there first. They remove the ribbon on the station to say it has been used, and other runners need to go elsewhere. Your task is to examine the distances that the runners take in going from THE TOP GATE to the BOTTOM GATE via a station.

YOU WILL SHOW HOW THE DISTANCE CHANGES DEPENDING ON THE STATION TO WHICH YOU HAVE TO SELECT TO RUN.



There are 18 stations along the fence line at 10 metre intervals.
The station closest to Corner A is 50 metres from Corner A.
The distances of the gates from the fence with the stations are marked on the map.

THE TASK
Investigate the changes in the total path length travelled as the runner goes from Gate 1 to Gate 2 after visiting one of the check point stations. To which station would the runner travel, if he/she wished to travel the shortest path length?

THE DIAGRAM – THE SCALED PLAN

Draw a scaled plan diagram of the section of the orienteering course (use graph paper). Scale is 1 cm to 10 metres.

You will need to mark the position of variables on the plan:
Distance 1, the distance from corner A to the stations,
Distance 2 is the remaining length on the base line, after *Distance 1* is marked.

THE CALCULATIONS

Find the total distance a runner travels.

For the station on the base line closest to Corner A,

Use Pythagoras' Theorem, to calculate the distance a runner travels going from the Gate 1 to the first station (the one closest to Corner A)

Calculate the distance a runner travels in going from the first station to Gate 2.

Calculate the total path length for the runner going Gate 1 – Station 1 – Gate 2.

Compare your result with that obtained from measurement of the path on the scaled diagram.

Repeat the calculations for the runner going to Station 2 and then repeat the calculations for the runners going to Station 3 and Station 4.

Does running via Station 1, or Station 2, or Station 3 make any difference to the overall length of the run?

MODEL FOR THE CALCULATIONS

List the sequence of mathematical steps (method of calculation) for finding the total distance a runner travels through the field.

THE LIST OF RESULTS OF TOTAL RUN DISTANCE

Use the Lists in your calculator to find the total distance across the field as 18 runners in the event go to one of the stations. Enter appropriate formulae to calculate the paths via each station. Record your results in a table, similar to the following.

Station Number	Distance from Corner A	Top Gate to Station Distance	Station to Bottom Gate Distance	Total Distance
1 2 3 4 And so on to 18				

GRAPH

Draw a graph that shows how the total run distance you run changes as you travel to the different stations (that is, as *Distance 1* increases).

Observe the graph, then answer these questions:

1. What shape is the graph? Does the graph look like a straight line?
2. Where is the station that has the shortest run total distance?
3. Could a 19th station be entered into the base line to achieve a smaller total run distance? Where would the position of the 19th station be?
4. Does the total distance run change at the same rate as you travel via station 1, or 2, or 3, or 4, …?
5. If you were the sixth runner to reach Gate 1, to which station would you probably need to travel?

THE ALGEBRA

What is the algebraic equation that represents the graph pattern?

Plot this graph of this equation on your graph of the points.

If you could put in a 19th station where would you put the station, and why?