

# CAS: TRAINER WHEELS FOR ALGEBRA USING THE TI89

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*Computer Algebra Systems (CAS) can perform routine algebraic manipulations quickly and correctly using inbuilt algorithms. An application called Symbolic Math Guide (SMG) can be added to a TI89 to enable the CAS to be used to help students learn how to perform step-by-step algebraic manipulation. SMG supports students when learning to “do the same to both sides” to simplify expressions and for solving equations. SMG provides “trainer wheels” for solving, enabling students to focus on the process of solving, hopefully improving their ability to work with algebraic expressions and equations. This paper will explore how SMG could be used in middle school algebra and discuss some approaches being used in the new RITEMATHS (Real World Problems and Information Technology) research project.*

## **Introduction**

Computer Algebra Systems (CAS) in the form of CAS calculators are very powerful and have the facility to work symbolically as well as graphically and numerically. Experience shows that pedagogical use (see for example Pierce and Stacey, 2002) of CAS by both teachers and students may assist students in developing their algebraic understanding, supporting those students who are experiencing difficulty and providing the option of extension activities for students who require more challenge in the curriculum (see for example Asp, Ball, Flynn & Stacey, 2002).

CAS calculators can perform routine algebraic manipulations quickly and correctly using inbuilt algorithms. With a CAS calculator there is the possibility to obtain answers for some questions with one line of entry, which is automatically simplified on the press of the ENTER button, or with just a single command. However total reliance on CAS is not our goal. If students have good algebraic insight (Pierce and Stacey, in press 2004) and basic manipulation skills it may even be more efficient for them to solve problems mentally or with a couple of lines of pen-and-paper working. Students still need such skills and effective use of CAS can be instrumental in assisting students to gain proficiency. One of the benefits of working with CAS is that students can be confident that each line of working will be executed correctly so they may learn from carefully observing the results of each command. In addition, instead of using a single multi-step command it is often possible to force the CAS to work step by step. While, with experience, students will also learn to condense the steps, it is important that these are transparent to students in their initial stages of learning. As teachers start to explore CAS calculators this potential of CAS to, for example, support equation solving by providing equivalent algebraic equations for each successive step of working becomes apparent. However, as we will illustrate below, forcing CAS to work step by step is somewhat cumbersome and often results in expressions which are not in the conventional form used when working with pen-and-paper only. Fortunately there are new possibilities for allowing students to work in an environment where the algebraic manipulation being carried out is correct, the steps are transparent and students are able to focus on selection of an efficient process for solving. One such software addition, Texas Instruments' Symbolic Math Guide (SMG), is described in this paper.

The inability to choose correct manipulations and, as a result write down equivalent expressions, when solving in a purely pen-and-paper environment, is a problem for many students. This can be alleviated with the use of CAS. For some students algebra is perceived to be an insurmountable obstacle and we believe that CAS may be used in a number of ways to provide some assistance for these students. In particular, access to newer software like SMG may provide “trainer wheels” for students learning algebra by providing options for each step, constraining students to generically applicable methods, and presenting students with step-by-step algebraic manipulation that is correct at each stage. This paper illustrates the use of SMG and discusses some of the features this facility adds to the ‘standard’ CAS.

## SOLVE and the HOME screen

We will begin by discussing why SMG is needed when the inbuilt SOLVE feature of a CAS calculator can be used to automatically find solutions for all the equations that students in years 9 and 10 normally encounter. Figure 1 illustrates use of the inbuilt SOLVE feature to solve a quadratic equation. For this typical example one line of entry and an appropriate choice from the menu produces a correct result. The values for  $x$  are given in surd form and the only peculiarity may be in the specific CAS calculator convention here of taking out a factor of negative one in the second value for  $x$ . Given that a CAS calculator can solve problems like that shown in figure 1, which is a fairly typical problem for students learning quadratics, it may seem that a CAS has little relevance in the years 9 and 10 curriculum, except for checking of answers. This is definitely not the case! A CAS can be very useful in helping students develop their understanding of algebra and may be used to improve the learning of algebra in the middle school curriculum.

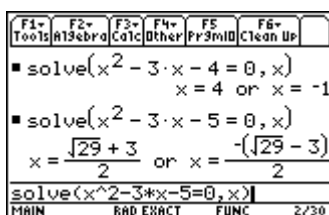


Figure 1

Example of solving quadratic equations using the inbuilt SOLVE feature

There are times when it is appropriate for teachers to encourage students to use the inbuilt SOLVE features of a CAS and there will be other occasions when teachers should discourage the use of this feature. Actively discouraging the use of the SOLVE feature, while still allowing CAS to be used, may provide opportunities to support the development of algebraic thinking by having students attempt to drive their CAS to do step-by-step solving in the HOME screen. For example, Figure 2 shows how a CAS can be used to solve a simple linear equation step-by-step. This technique parallels conventional working with pen-and-paper except for the way in which the CAS records the inputs and the advantage that each step suggested by the student will be carried out following correct mathematical logic. Errors will not be masked by incorrect algebraic manipulation.

In this case “doing the same to both sides” enables students to plan each step of working with the consequences of intermediate steps being visible. Students need to be able to interpret the CAS display where both inputs and outputs are displayed. Some of these will resemble pen-and-paper working, while other steps are displayed differently, for example ‘subtracting 3 from both sides is indicated by the line  $(2 \cdot x + 3 = 7) - 3$ .

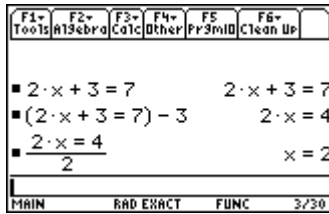


Figure 2

Example showing 'steps' from main menu

Use of the HOME screen when teaching the processes of beginning algebra is still limited by the lack of transparency of all intermediate steps even when performing step-by-step calculations. Most intermediate steps are not shown when inbuilt CAS algorithms simplify expressions. Figure 3 shows an example which illustrates how inbuilt algorithms may simplify expressions without showing the details of the working.

In each of the two examples below, once the initial equation has been keyed in, and the ENTER button pressed, the inbuilt algorithms automatically simplify the left hand side of the equation. Students learning algebra may not see the intermediate steps of  $2x-2+3=11$  in the first example, or the link of the common denominator in the second example.

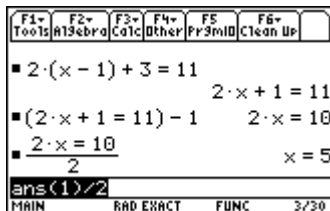


Figure 3a

Example showing automatic simplification

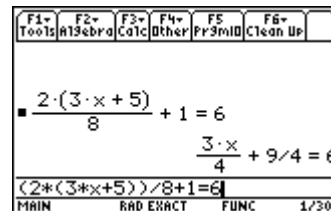


Figure 3b

Example showing automatic simplification

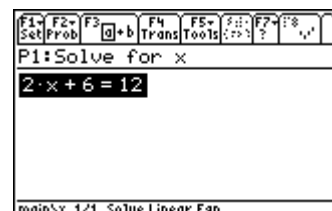
In these cases the CAS will perform automatic simplification but for students who experience difficulty with algebra and/or fractions the equivalence of successive equations may not be obvious. Even when an attempt is made to force a CAS to do step-by-step algebra by “doing the same to both sides” it is likely that a CAS calculator will simplify at particular stages and that the procedures which have been performed may not be obvious to students who are learning algebra. When students are confident and competent with algebraic manipulation the issue of intermediate steps being visible is less problematic as students will hopefully be able to perform one or two steps mentally and hence readily identify equivalent equations. The examples discussed in the next section demonstrate that for the purpose of assisting beginning algebra students the downloadable application Symbolic Math Guide(HREF1) is extremely helpful.

## Symbolic Math Guide

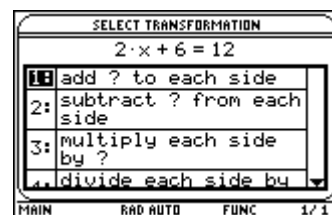
Symbolic Math Guide (SMG) enables students to work symbolically using a CAS calculator, in this case a TI89, to perform step-by-step algebraic manipulation. SMG still involves some automatic simplification but this occurs as a one-step process rather than applying inbuilt algorithms that perform a number of steps at once but in a manner which is hidden from the operator. Essentially SMG requires students to choose appropriate intermediate steps when working with equations and expressions. SMG supports students learning to simplify expressions and privileges "doing the same to both sides" as the method for solving linear equations. SMG provides optional 'trainer wheels' for simplifying and solving by supplying a menu of possible operations. These suggestions prompt the student who does not know how to proceed. It is anticipated that having developed the skills of solving with the technology, and following experiences of correct working and solutions, students will be able to solve simple problems without technology. SMG is one of the technologies being explored in the new RITEMATHS (Real World Problems and Information Technology) research project being jointly conducted by university researchers and practicing teachers.

To illustrate some of the affordances of SMG the following examples, shown in figures 4 and 5, illustrate how it may be used to support step-by-step solution of linear and quadratic equations. The first example, figure 4, shows a process for solving  $2x+6=12$  for  $x$  and mimics the types of steps that teachers might expect in a pen-and-paper solution.

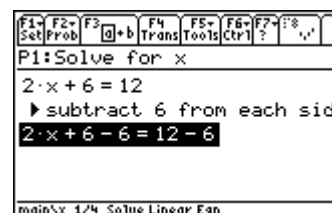
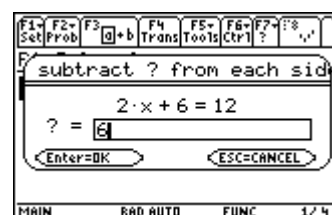
The first step in using SMG requires students to correctly key in the equation to be solved and indicate that they are solving for  $x$ . Students should be encouraged to find an efficient path to the solution, however, SMG will enable students to find a solution even when an inefficient method is chosen.

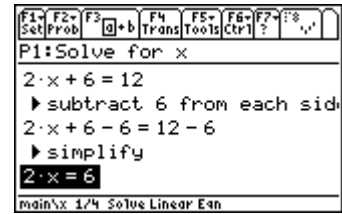


Students may enter an operation or choose from the 'trainer wheels' menu. While it is perfectly correct to choose to add 6 to both sides (as some students will do) giving  $2x+12=12$ , this is not an efficient method for moving towards the equivalent equation  $x=3$ . The challenge for students is to minimise the number of steps, thus finding an efficient path for the solution of the equation.

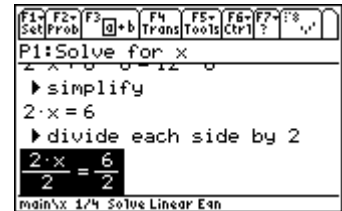


Students can now just subtract six from both sides. SMG shows this step and then students need to press ENTER before the simplified version of the equation is given. This process supports the development of an understanding that for solving an equation students must "do the same to both sides". Notice that SMG shows both a verbal description of the process being performed as well as the symbolic representation of these mathematical steps.





Students may then divide each side of the equation by two and once again SMG shows the intermediate step before simplifying. The application does not perform automatic simplification at this stage and this ensures that students can see that both sides of the equation are being divided by two. Once the ‘simplify’ prompt appears, pressing the ENTER button will produce the modified version..



An additional SMG feature enables students to ‘check’ their answer. This feature automatically substitutes the value for  $x$  into the original equation to show that the left hand side is indeed equal to the right hand side of the equation.

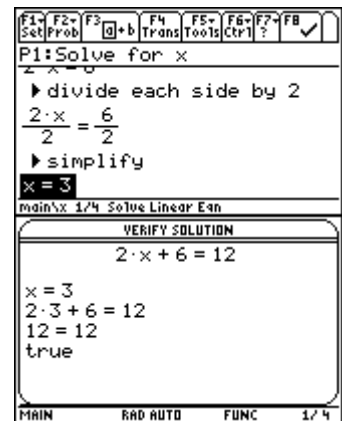


Figure 4  
Example showing solution of linear equation using SMG

The equation solved above could have been solved in a number of different ways other than by the method that was selected. One constructive possibility for classroom discussion is to compare the efficiency of different choices and of the order of particular steps. One strength of SMG is that each step suggested by the student will be correctly executed, so students are seeing technically correct algebraic calculations at each step. This assists in overcoming the problem which occurs when a student is unable to write down the successive equivalent equations required when solving an equation. Since SMG allows students to focus on the solution process, a challenge in the classroom could be to solve a given problem in the least number of steps.

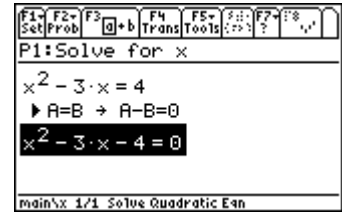
SMG constrains students to proceeding in single steps which they may come to see as tedious. Anecdotal evidence suggests that, after some of this guided practice, students choose to solve simple equations either mentally or with a small number of pen-and-paper steps. Using the ‘trainer wheels analogy’ students are able to remove the ‘trainer wheels’ and ride their bike, in this case solve simple equations, independently.

We will now consider an example demonstrating the use of SMG to solve a quadratic equation. Again the use of SMG allows students to focus on the new solution process rather than the ‘calculations’. Here it is assumed that students can factorise a quadratic expression.

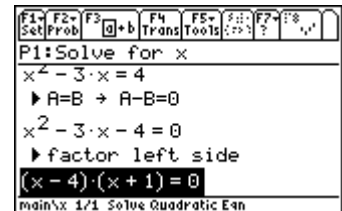
First students must enter the equation to be solved.



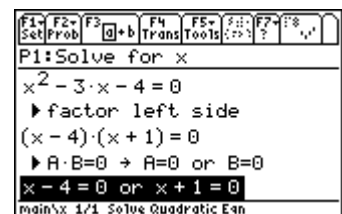
Students now need to recognise that in order to find solution values for  $x$  they will need to transform the equation to the format  $f(x)=0$ . (Perhaps by linking equation solutions with graph zeros). This option must be selected from the given menu options. Note that the SMG programming shows the general case and hence students are required to recognize the general form of the equation that they are solving.



The left hand side can be factorised. While SMG will perform the factorisation, students must recognise that factorisation is required. The resulting output shows the correct factorisation of  $x^2-3x-4$ .



Students must then recognise that application of the null factor law will give two linear equations to solve.



The linear equations can then be solved in one step or the student could continue and solve each linear equation separately with step by step guidance from SMG.

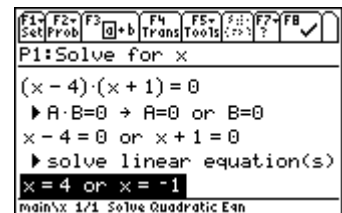


Figure 5  
Example showing solution of quadratic equation using SMG

The example above shows that the SMG ‘hints’ menu reminds students of the likely steps required in the process for solving a quadratic equation. SMG is able to be used for a variety of equations and can be utilized to help students develop their ability to deal with algebraic expressions and equations in a number of year levels.

## Conclusion

The examples in this paper give an indication of how the use of SMG can enable a powerful CAS calculator to assist students in developing their skills in working with algebra. Step-by-step guided algebra software, such as SMG, may make algebra accessible to more students. The ability to tackle problems in the knowledge that algebraic manipulation will be correct will hopefully result in more students having increased mathematical confidence.

Repeated experience with the assistance of the option menus provided will help students remember processes for dealing with algebraic expressions and equations. A greater exposure to correct algebraic working should help more students make better progress in algebra. Careful observation of many correctly set out and correctly executed options at each stage may help students learn to perform these routines for themselves either mentally or using pen-and-paper. Careful guided practice with ‘trainer wheels’ may launch more students to independent implementation of algebraic methods and an understanding of the processes behind the automatic solutions provided by CAS in the standard HOME symbolic mode.

## References

Asp, G., Ball, L., Flynn, P., & Stacey, K. (2002). Expanding algebra by substituting CAS. In C. Vale, J. Roumeliotis, & J. Horwood (Eds.), *Valuing Mathematics in Society. Proceedings of the 39th Annual Conference of the MAV* (pp. 348-364). Melbourne: Mathematical Association of Victoria.

HREF1

<http://education.ti.com/us/product/apps/smg.html> - Symbolic Math Guide, Texas Instruments Pty Ltd

Pierce, R., & Stacey, K. (2002). Monitoring effective use of computer algebra systems. In B. Barton, K.C. Irwin, M. Pfannkuck, & M. O. J. Thomas (Eds.), *Mathematics Education in the South Pacific. Proceedings of the 25th Annual Conference of the Mathematics Education Research Group of Australasia* (pp. 575-582). Sydney: MERGA.

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