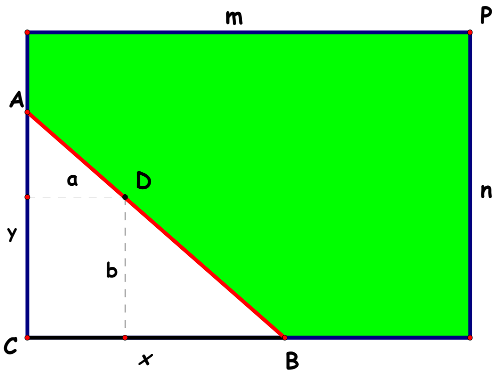
Cutting Corners Post Problem – Shortest Fence



A farmer wants to fence off a corner of his m metres by n metres rectangular paddock by making use of a post (D) located a metres from one side of the paddock and b metres from the other side.

Where should the farmer locate the corners (A and B) of the new fence so that the shortest length of fence is used?

1. Open the file PostProbMinLengthFence.gsp. Maximise the size of the file on the screen and click on the tab labelled **Problem**. Experiment by dragging the **points P, D, and B** to get a sense of the problem and an understanding of what is given (and how it can be changed) and what is to be found.

2. Click the tab labelled **Function of x Rule**.

   Click the button to set **P** to a position so that: **m is 9.00** and **n is 11.30**
   Click the button to set **D** to a position so that: **a is 2.80** and **b is 4.20**

   These are the given values for m, n, a and b for the problem. Other values could have been chosen.

   a. Click the button **Show point (x,d)**. Drag point **B** to change x and observe how the point **(x,d)** moves as x changes. Change x until you think the point **(x,d)** is at it's lowest position. Record the values of x and d.

   x = _________ d = _________

   b. Click the button **Show locus of point (x,d)**. Drag point B to change x and observe how the point **(x,d)** moves along the locus as x changes. Change x until you think the point **(x,d)** is at it's lowest position. Record the values of x and d.

   x = _________ d = _________
   [These answers might be more accurate than those found in part (a).]

   A new fence post should be located about __________ metres to the right of point C in order to give the shortest length fence.

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c. Click the button **Show Function of x Plot**. Drag point B to change x and again observe how the point **(x,d)** moves as x changes. Note that the point **(x,d)** does not move along the entire function plot.

Change x until you find what appears to the smallest value of x for the point **(x,d)**?

The smallest value of x for the point **(x,d)** is about ____________

Change x until you find the largest value of x for the point **(x,d)**?

The largest value of x for the point **(x,d)** is ____________

d. Use similar triangles and the Theorem of Pythagoras to find the rule for the **Function of x Plot**. Show and explain the steps in your working below. If you want to see the function rule click the button **Show Function of x Rule**.

3. Use the function rule from 2(d) to graph the function with **a graphing or CAS calculator** and then use the calculator to find a more accurate answer to the problem for the set values of m, n, a and b. Record your answers below.

x = ________ d = ________

The new fence posts should be located about _______ metres to the right of point C and about _______ metres above point C for the smallest area lost.

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4. Return to the file PostProbMinLengthFence.gsp and click the tab **Min of Function.** In this sketch the minimum point for the graph of the **Function of x Rule** is shown as **FMin**.

   Click the button to set **P** to a position so that: **m is 9.00** and **n is 11.30.**

To answer the following questions, click on points **D** or ***B*** and use the right [→] and left [←] keys and the up [↑] and down [↓] keys to change their position.

a. Find a position for the post so that the shortest fence is not the Min of the Function but the shortest fence is found at (what appears to be) the smallest value of x for the point (x,d).

   Post Position: a = ________ b = ________

   x = _______ d = ________

b. Find a position for the post so that the shortest fence is not the Min of the Function but occurs at the largest value of x for the point (x,d).

   Post Position: a = ________ b = ________

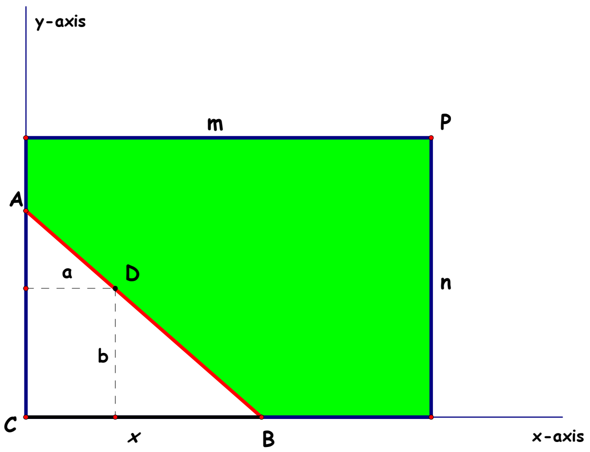
   x = _______ d = ________

The above results for question (4) tell us that the location of the post in the paddock can sometimes make the shortest length fence be something other than the minimum of the function, but the shortest length fence is still one of the values of the function.

Another Way to Solve the Problem

5. Consider point C to be the origin of a coordinate system with CB on the x-axis and CA on the y-axis. Then the point D has coordinates (a, b).

   Let k be the gradient of the line that passes through the points A, B and D.



a. Click the tab **d as function of k.** This sketch shows the **length of the fence d as a function of the gradient k.** Drag point B to change the gradient k.

   What is the smallest value of k that is possible? ______________

   What is the largest value of k that is possible? ______________

b. Click the button **Show locus of point (k,d).** Drag point B to change k and observe how the point **(k,d)** moves along the locus as k changes. Change k until you think the point **(k,d)** is at it's lowest position. Record the values of k and d.

   k = _________ d = _________

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c. k is the gradient of the line joining the points A and B that also passes through point D. Find the equation of this line in terms of k, a and b. Show your work below.

d. Use the equation you found in part (c) to find the coordinates of the points A and B in terms of k, a and b. Show your work below.

e. Use your answers to part (d) to write a function rule for the length, d, of AB in terms of k, a and b.

f. Let k = x and rewrite the rule you found in part (e) in terms of x.

g. Return to the page **d as function of k** in the file PostProbMinLengthFence.gsp. Choose **Plot New Function** from the Graph Menu and plot the function rule that you found in part (f) using the values for a and b shown on the Sketch.

If the **locus for the point (k,d)** does not form part of the graph of this function, check and redo your work above from (c) to (f) until the locus does form part of the plotted function.

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6. Use the correct function rule from 5(f) to graph the function with **a graphing or CAS calculator** and then use the calculator to find a more accurate answer to the problem for the set values of m, n, a and b. Record your answers below.

k = ________ d = ________

The new fence posts should be located about _________ metres to the right of point C and about _________ metres above point C in order to give the shortest length fence.

Explain and show what you did to find these values.

7. If you have a **CAS calculator**, use the correct function rule from 5(d) together with the calculator's **fMin** operation (will give k that makes **d as function of k** a minimum) and **algebra**, to **find the answers as rules in terms of a, b and k**. Record the rules below.

k = ____________ d = ____________

The new fence posts should be located ______________ metres to the right of point C and ______________ metres above point C in order to give the shortest length fence.

Explain and show what you did to find these rules.

Note: The location of the post in the paddock can sometimes make the shortest length fence be something other than the minimum of the function, so the above rules may not always give the answer to the problem.

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