Post Problem for Shortest Length of Fence
(Using CAS)

Method 1

We will set the dimensions of the paddock to be m = 9 metres and n = 11.3 metres and locate the post at a = 2.8 metres and b = 4.2 metres. Other values could be chosen. **For this method CAS <u>cannot solve</u> the problem in general terms.**

Let *x* be the distance from B to C. Let *y* be the distance from A to C. See the diagram on Student Problem Sheet or GSP Sketch for this problem.

Let d be the distance from A to B. Using Pythagoras we have:

$$d^2 = x^2 + y^2$$

Solving this equation for *d* we have:

$$\text{Solve}\left(d^2 = x^2 + y^2, d\right) \mid d > 0 \qquad d = \sqrt{x^2 + y^2}$$

Using similar triangles we have:

$$\frac{x}{y} = \frac{x - 2.8}{4.2}$$

Solving this equation for *y* gives *y* in terms of *x*:

$$\text{Solve}\left(\frac{x}{y} = \frac{x - 2.8}{4.2}, y\right) \qquad y = \frac{4.2 \cdot x}{x - 2.8}$$

Substituting this result for *y* in the equation for *d* from Pythagoras and defining *d* as a function of *x* gives:

$$\text{Define } f(x) = \sqrt{x^2 + y^2} \mid y = \frac{4.2 \cdot x}{x - 2.8} \text{ and } x > 2.8 \qquad \text{"Done"}$$

$$d = f(x) \qquad d = \frac{x \cdot \sqrt{x^2 - 5.6 \cdot x + 25.48}}{x - 2.8}$$

Finding x that gives the minimum of *d*:

$$\text{fMin}\left(f(x), x\right) \mid x > 2.8 \text{ and } x < 9$$

$$x = 6.46904$$

Warning: Questionable accuracy

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Substituting this value of *x* into *f(x)* gives the minimum length of fencing required:

$$d = f(x) \mid x = 6.46904$$

$$d = 9.83288$$

Since x = BC, then BC = 6.46904. Using this value for x, the distance from A to C, which is y, can be found:

$$y = \frac{4.2 \cdot x}{x - 2.8} \mid x = 6.46904$$

$$y = 7.4052$$

So the farmer should locate the corners of the fence about 6.46904 metres to the right of AC and about 7.4052 metres above BC when the post's location is given by a = 2.8 metres and b = 4.2 metres.

The above method may give the solution to the problem, but it may not, because the location of the post in the paddock may make the minimum for the function d(x) occur at an end point of the function's domain (which is determined by the location of the post) and this domain may not include the value of x found by using fMin.

Method 2 - Finding a General Solution to the Problem

For **Method 2** consider point C to be the origin of a coordinate system with CB on the x-axis and CA on the y-axis. ***CAS can solve the problem with Method 2 in general terms.***

The post is located at (a,b) and any line through this point with gradient k is given by y = k(x-a) + b.

The x-intercept and y-intercept of this line are:

$$y = k \cdot (x - a) + b \mid x = 0$$

$$y = b - a \cdot k$$

$$y = k \cdot (x - a) + b \mid y = 0$$

$$0 = k \cdot x - a \cdot k + b$$

$$\text{solve}(0 = k \cdot x - a \cdot k + b, x)$$

$$x = \frac{a \cdot k - b}{k}$$

The hypotenuse of the right triangle ACB is the length of the fence and its length is given by the function found by finding the distance between the two points A (0, b-ak) and B ((ak-b)/k, 0).

$$\text{Define } d(k) = \sqrt{(b - a \cdot k)^2 + \left(\frac{a \cdot k - b}{k}\right)^2} \mid a > 0 \text{ and } k < 0 \text{ and } b > 0$$

"Done"

$$d(k)$$

$$\frac{(a \cdot k - b)\sqrt{k^2 + 1}}{k}$$

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Next find the value of k, that makes d(k) a minimum, and use this to find the minimum for d(k):

$$\mathrm{fMin}(d(k), k)$$

$$k = \infty \text{ or } k = -\infty \text{ or } k = \frac{-\left(\sqrt[3]{b}\right)}{\sqrt[3]{a}} \text{ or } k = 0$$

$$d(k) \mid k = \frac{-\left(\sqrt[3]{b}\right)}{\sqrt[3]{a}}$$

$$\left(a^{2/3} + b^{2/3}\right)^{3/2}$$

Thus the minimum length is given by the rule:

$$minlength = \left(a^{2/3} + b^{2/3}\right)^{3/2}$$

The location of the fence corners for this minimum length fence are given by these rules:

$$y = b - a \cdot k \mid k = \frac{-\left(\sqrt[3]{b}\right)}{\sqrt[3]{a}}$$

$$y = a^{2/3} \cdot \sqrt[3]{b} + b$$

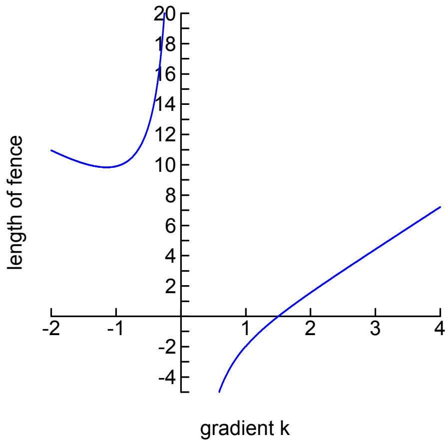
$$x = \frac{a \cdot k - b}{k} \mid k = \frac{-\left(\sqrt[3]{b}\right)}{\sqrt[3]{a}}$$

$$x = \sqrt[3]{a} \cdot \left(a^{2/3} + b^{2/3}\right)$$

The above may give the solution to the problem, but may not, as the location of the post in the paddock may make the minimum for the function d(k) occur at an end point of the function's domain (which is determined by the location of the post) when this domain does not include the value of k given by:

$$k = \frac{-\left(\sqrt[3]{b}\right)}{\sqrt[3]{a}}$$

In the case when a = 2.8 metres and b = 4.2 metres we have for the graph of d as a function of k:



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Calculating the results using the general rules found above we have:

$$k = \frac{-\left(\sqrt[3]{b}\right)}{\sqrt[3]{a}} \mid a = 2.8 \text{ and } b = 4.2 \qquad k = -1.14471$$

$$minlength = \left(a^{2/3} + b^{2/3}\right)^{3/2} \mid a = 2.8 \text{ and } b = 4.2$$

$$minlength = 9.83288$$

$$y = a^{2/3} \cdot \sqrt[3]{b} + b \mid a = 2.8 \text{ and } b = 4.2$$

$$y = 7.4052$$

$$x = \sqrt[3]{a} \cdot \left(a^{2/3} + b^{2/3}\right) \mid a = 2.8 \text{ and } b = 4.2$$

$$x = 6.46904$$

These answers are the same as those found when using Method 1 above.

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