Post Problem for Smallest Area Lost
(Using CAS)

CAS can solve this problem in the general terms of *a* and *b* where $n > b > 0$ and $m > a > 0$.

Let *x* be the distance from B to C. Let *y* be the distance from A to C. See diagram in GSP Sketch.

The area lost from the paddock is given as follows:

$$A = \frac{1}{2} \cdot x \cdot y$$

Using similar triangles we have:

$$\frac{x}{y} = \frac{x - a}{b}$$

Solving this equation for *y* gives *y* in terms of *x*:

$$\text{Solve}\left(\frac{x}{y} = \frac{x - a}{b}, y\right) \qquad y = \frac{b \cdot x}{x - a}$$

Substituting this result for *y* in the equation for *A* and defining *A* as a function of *x* gives:

$$\text{Define } f(x) = \frac{1}{2} \cdot x \cdot y \mid y = \frac{b \cdot x}{x - a} \text{ and } x > a \text{ and } a > 0 \qquad \text{"Done"}$$

$$Area = f(x) \qquad area = \frac{b \cdot x^2}{2 \cdot (x - a)}$$

Finding the minimum of *area* gives:

$$\text{fMin}(f(x), x) \mid x > a$$

$$x = \infty \text{ or } x = -\infty \text{ or } x = 2 \cdot a \text{ or } x = 0 \text{ or } b = 0$$

Knowing that $a < x < n$ and $a > 0$, the minimum of the function occurs when $x = 2a$. This gives the location of the horizontal corner post. Then, the location of the vertical corner post is given by:

$$y = \frac{b \cdot x}{x - a} \mid x = 2 \cdot a \qquad y = 2 \cdot b$$

Substituting $x = 2a$ into *f(x)* gives the minimum area lost.

$$minarea = f(x) \mid x = 2 \cdot a$$

$$minarea = 2 \cdot a \cdot b$$

Note that the location of the post in the paddock may result in the the smallest area lost being other than the minimum of the function, due to constraints imposed on the domain for the area function. When this happens the minimum area is not given by 2ab.

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