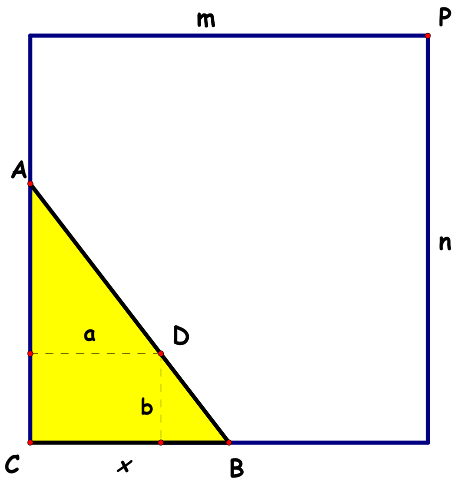
Cutting Corners Post Problem - Smallest Area Lost



A farmer wants to fence off a corner of his m metres by n metres rectangular paddock by making use of a post (D) located a metres from one side of the paddock and b metres from the other side.

Where should the farmer locate the corners (A and B) of the new fence so that the smallest area is lost from his existing paddock?

1. Open the file PostProbMinAreaLost.gsp. Maximise the size of the file on the screen and click on the tab labelled **Problem**. Experiment by dragging the **points P, D, and B** to get a sense of the problem and an understanding of what is given (and how it can be changed) and what is to be found.

2. Click the tab labelled **Function of x Rule**.
   a. Drag **P** to a position so that:

      m is between 8 and 10 and n is between 10 and 12

      Then without moving **P** again, move **D** to a position so that:

      $2a < m$ and **$2b < n$**

      Record your values:

      m = __________ n = __________ a = __________ b = __________

      These are the given values for m, n, a and b for the problem.

   b. Click the button **Show point (x,A)**. Drag point **B** to change x and observe how the point **(x,A)** moves as x changes. Change x until you think the point **(x,A)** is at it's lowest position. Record the values of x and A.

      x = _________ A = _________

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c. Click the button **Show locus of point (x,A)**. Drag point B to change x and observe how the point **(x,A)** moves along the locus as x changes. Change x until you think the point **(x,A)** is at it's lowest position. Record the values of x and A.

x = _________ A = _________

These values might give a 'better answer' to the problem than those in part b. Why?

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d. Click the button **Show Function of x Plot**. Drag point B to change x and again observe how the point **(x,A)** moves as x changes.

Does the point **(x,A)** move along the entire function plot? Should it? Why?

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e. Use algebra to find the rule for the **Function of x Plot**. Show and explain the steps in your working below. (Hint: Make use of similar triangles.) If you want to see the function rule click the button **Show Function of x Rule**.

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3. Use the function rule from 2(e) to graph the function with **a graphing or CAS calculator** and then use the calculator to find <u>a more accurate answer</u> to the problem for the values of m, n, a and b that were recorded in question 1 above. Record your answers below.

x = _________ A = _________

The new fence posts should be located about ________ metres to the right of point C and about ________ metres above point C for the smallest area lost.

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4. If you have a **CAS calculator**, use the calculator's **fMin** operation (will give x that makes the Function of x a minimum) and **algebra** to show that the smallest area lost may be given by 2ab. Show and explain what you did below.

5. Return to the file PostProbMinAreaLost.gsp and click the tab **Min of Function.** In this sketch the minimum point for the graph of the **Function of x Rule** is shown as **FMin**. To answer the following questions, click on points **D** or ***B*** and use the right $\boxed{\rightarrow}$ and left $\boxed{\leftarrow}$ keys and the up $\boxed{\uparrow}$ and down $\boxed{\downarrow}$ keys to change their position.

   a. Find a position for the post so that a corner of the fence that gives the smallest area lost is at the top left-hand corner of the paddock AND the smallest area lost is not the Min of the Function.

      Post Position: a = __________ b = __________ Area Lost = __________

   b. Find a position for the post so that a corner of the fence that gives the smallest area lost is at the bottom right-hand corner of the paddock AND the smallest area lost is not the Min of the Function.

      Post Position: a = __________ b = __________ Area Lost = __________

   c. Find a position for the post so that one corner of the fence that gives the smallest area lost is at the top left-hand corner AND the other is at the bottom right-hand corner of the paddock AND the smallest area lost is not the Min of the Function.

      Post Position: a = __________ b = __________ Area Lost = __________

The above results for question (5) tell us that the location of the post in the paddock can sometimes make the smallest area lost be something other than 2ab. That is, the smallest area lost will always be a value for the Function of x Rule, but it may not be the minimum for this function, which is 2ab.

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