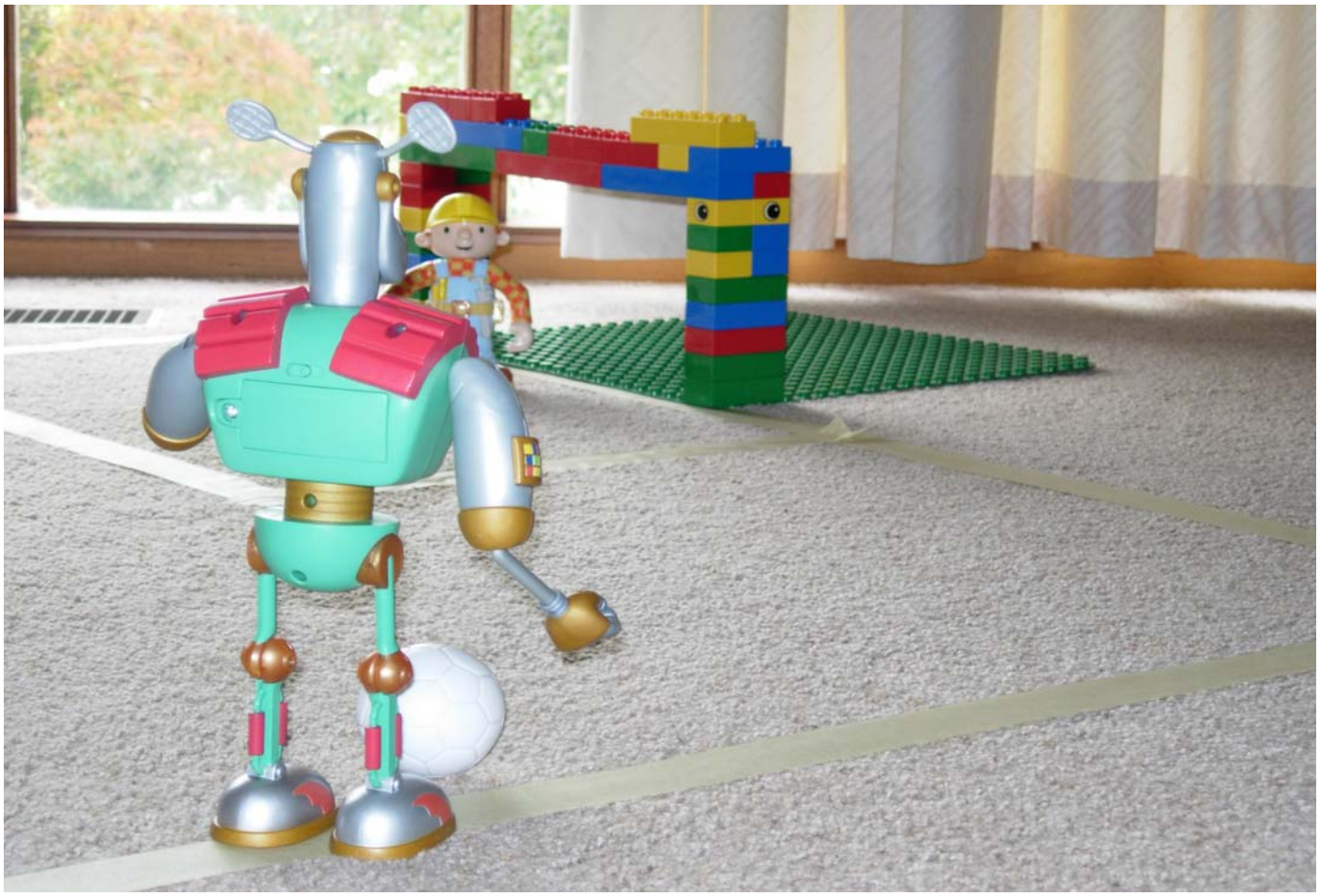


# SHOT ON GOAL

**Name:** _____

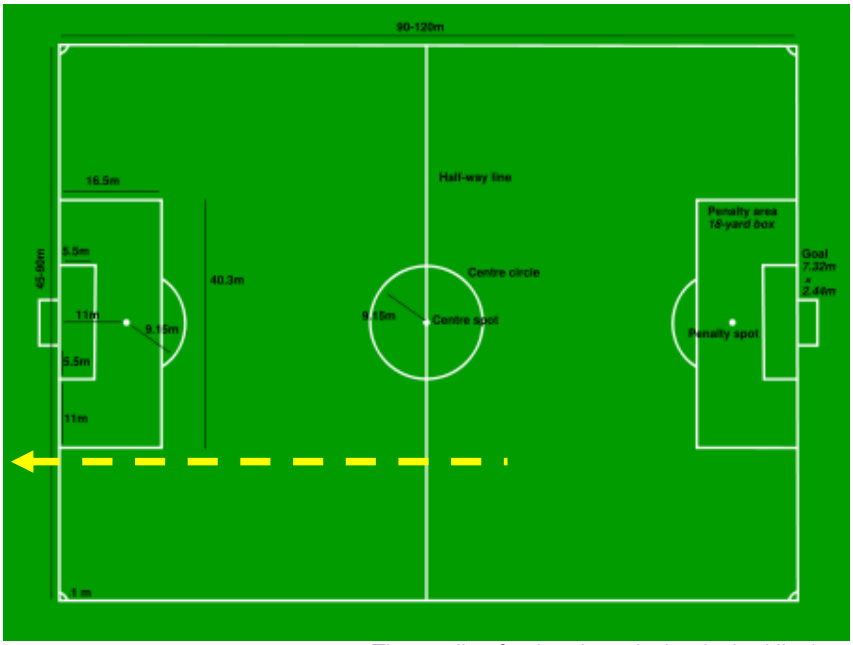
*Football – scoring goals and trigonometry*

introducing
TI-nspire™ CAS

## THE TASKS

You are an assistant coach with the local football team, the Luther Lions FC. At a training session, your task is to demonstrate to the young players the zones on the football pitch which show the greatest opening between the goal posts. This knowledge will assist forwards in knowing when to shoot for goal as the angle of the goal opening is greatest and the defenders in revealing which parts of the ground need to be controlled.

## VISUALIZE THE SITUATION




(The run line for the player is the dashed line)

There is only 5 minutes remaining on the clock until the end of the game. Your team has the ball. Jonathan Angle, your star forward, makes a run (dotted line). He has broken clear from the defenders. His path is on a line parallel to the side touch lines and 15m to the left of the near post. He unleashes a powerful left-foot shot on goal.

## THE QUESTION

Has Jonathan kicked the ball from the position giving the maximum angle for opening up the goal thereby increasing the chance for scoring a goal?

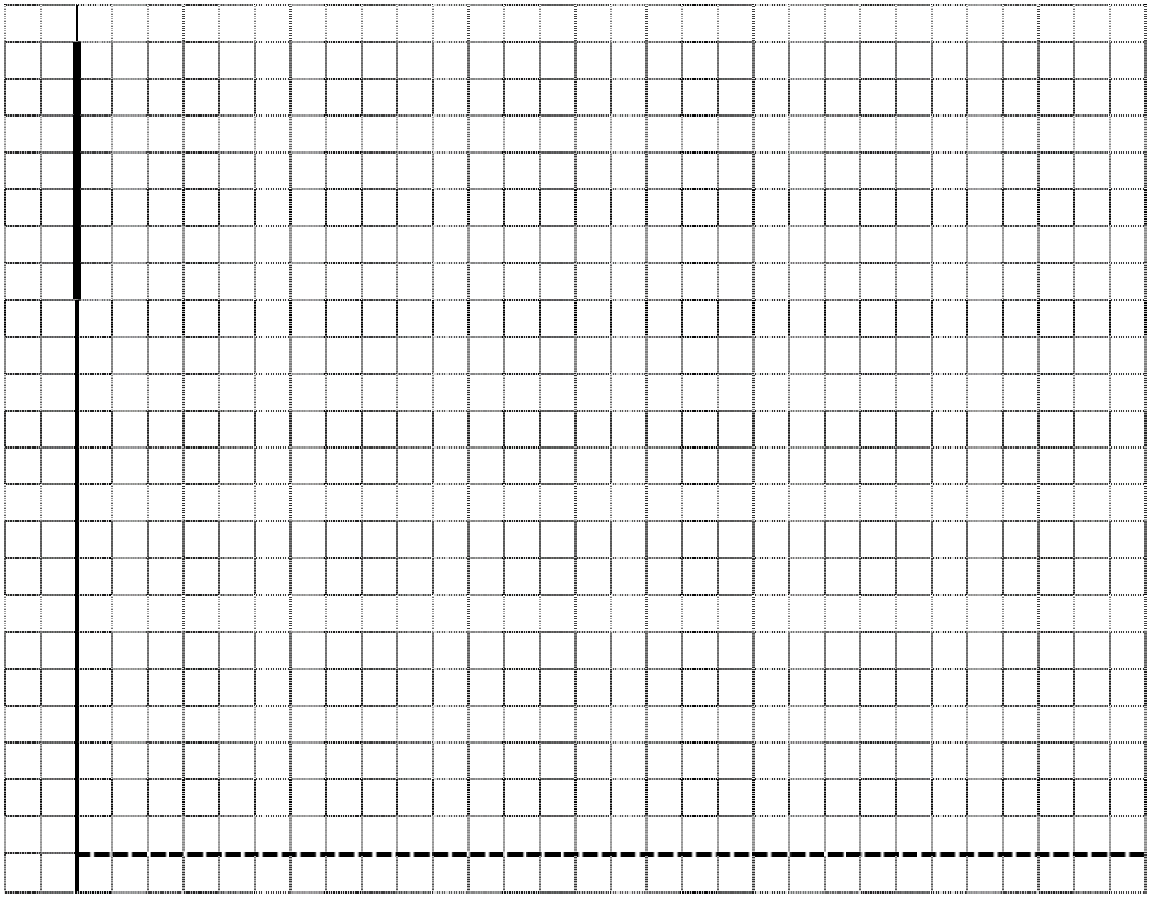
 

# PART 1

## 1 .1 Clarifying the Idea – Drawing the Diagram

A football game is a 3-D situation. You require a 2-D dimensioned PLAN of the field to show the mathematical relationships of the angle from the kicking spot to the goal posts.

1. Your diagram should be completed on the GRAPH PAPER below.
2. The football field shows the positions of the **Goal Line**, the **Touch Line**, and the **Run Line.**
3. Mark a **Spot** from which the shot could have been taken and the **Angle formed** with the goal.
4. On your diagram clearly mark all dimensions. Mark the right angles, and angles required for the calculation of the shot on goal angle. The vertices need to be labelled (A, B,C …) to allow you to refer to lines and angles in your work.



## 1.2 Clarifying the Idea – Determining the Angle

You are to calculate the size of the angle formed from the kicking spot to the near and far posts of the goals.

1. Record the position of 4 different kicking spots on the Run Line which is 15m to the left of the goal posts.
2. Find the Angle of the Shot from each of these kicking spots. (Four fully worked manual calculations to determine the angle of the shot on goal are required.)

## 1.3 Clarifying the Idea – Thinking about the Problem

How do the results of the calculations suggest that there is a point for a best shot on goal ?

# PART 2

## 2.1 Generalize the Situation – Capturing new data

Using technology to replicate the manual steps

You have done 4 manual calculations for the values of the angle for the shot on goal. We now will capture more information to illuminate how the values of angle for the shot on goal change as you approach the goal touch line.

### CONSTRUCTION OF A DYNAMIC GEOMETRY APPLICATION

Open a Graph and Geometry application

1. Set the scales on the axis to cover at least one quadrant of the football field

2. Hide the scales, but reveal the grid. On this grid, construct a dynamic geometry application of the shot on goal. The application should
   a. Show the goal touch line with the goal posts P and T. The distance between P and T is 7.3 metres.
   b. 15 metres to the left of left post, construct the run line SH, where S is the position from where the shot is taken.
   c. The ∟PST is the shot on goal angle.
   d. Measure the distance SH and the ∟PST.

e. Open the variable menu, and link the variable **'d'** to the distance SH and the variable **'s'** for ∟PST.

Open a List and Spreadsheet application

3. Set up the manual capture equation the values for the distance along run line and the angle for the shot on goal.

4. Capture a record of the 30 values for the angle of the shot on goal and the distance from the Goal touch line to the kicking spot along the run line.

Open a Graph and Geometry application

5. Draw a scatter plot of the shot on goal angle (y) as a function of the distance along the run line (x).

## 2 .2 Generalize the Situation – Interpreting the Data

Searching for the maximum goal opening

a) Trace along the points plotted on the graph. Determine the highest value for the shot on goal angle. What is the value? What distance is the kick from the goal touch line?

b) Examine on your table. Between what two kicking spot distances do you conclude that the maximum angle for the shot on goal must happen?

c) If you change the distance the run line is from the near post (from 15 metres to 5 metres), what effect do you think there would be on the shot on goal angle? ? If you were a defender how would employ this knowledge in marking an opposition player with the ball?

## 2 .3 Generalize the Situation – Modelling the Data

Your task is to find an algebraic model to fit your graph and calculations. One method of discovering an algebraic model is to generalise what you did when you did manual calculations.

A. A clear idea of the purpose of the investigation

1. What is it that I am trying to discover?

B. A knowledge of the relationships

1. What factors control the changes in the value I am trying to discover?

2. What values remain constant in the investigation?

3. What mathematical ideas / rules / procedures show how these values are related? Are the values directly related, or do you need to find transitional values?

### THE EQUATION FOR THE EVENT.

What is the algebraic equation that represents pattern revealed in the graph?

Hints

1. Observe what you did in the manual calculations.
2. Imitate the steps of the solution of problem using variables not numbers
   - Keep a list of the variable names and what they represent.
   - Some variables have values that do not alter in any situation. Do you have any variable of this type? If you do, then you can replace the variable with this value.
   - Some variables change only in different scenarios. Do you have any of these types of variables.
   - Other variables change continually throughout the investigation.
3. The final equation is a statement of what you are trying to find, using the values that are causing the observed changes.

# PART 3

## 3 .1 New insights – Checking the Model

Draw the graph of your mathematical model on the scatter plot of the distance – angle size values.

1. How does the graph of your model confirm / reject your equation?

2. What is the exact position on the field where the maximum angle for the shot on goal occurs for your run line? Explain how you worked out the value of this maximum angle.

## 3 .2 New insights – Checking the Model

Investigate how the position for the kicking spot of the maximum angle of the shot on goal varies as the run line moves nearer to the near goal post.

This investigation can be conducted either

1. EXPERIMENTALLY and GRAPHICALLY.

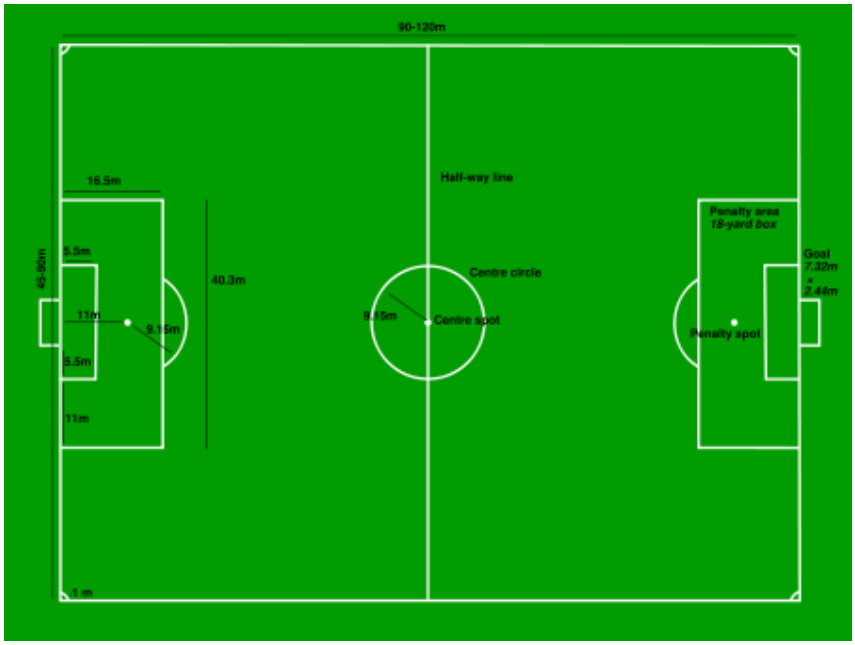
This will require you to collect data (using the applet) and plotting results (run line distance from post and distance to spot on run line) to reveal any the patterns in placement of spots for the maximum angle. Then, establish the equation.

2. EXTENDING THE ALGEBRAIC PROCEDURES.

This will require you to introduce a variable for distance of the run line is from the near post. Then, establish the equation using the power of CAS to handle the mathematical computations for the analysis.

# The Dimensions of a FOOTBALL FIELD




Goal opening = 7.32 metres (8 yards)

Length = 90 to 120 metres

Width = 45 to 90 metres (60 yards)