LEARNING ABOUT PARABOLAS AND TRANSFORMATIONS

WITH

GEOMETER'S SKETCHPAD

AND

A GRAPHING CALCULATOR

Name ______________________________

Teacher ___________________________

Activity 1 – Dilations and Reflection

For this activity you will use *Geometer's Sketchpad* to help see what happens to the graph of $y = ax^2$ when the number a is changed.

1. The basic quadratic relationship is given by the rule $y = x^2$.

   What is the value of a in $y = ax^2$ for the basic quadratic? a = _____

2. Click the button [Dilations and Reflection].

   The graph of $y = x^2$ is a parabola and it is drawn in red.

   Give the following key features of the graph of $y = x^2$.

   Coordinates of the turning point (_____ , _____)

   Coordinates of the x-intercept (_____ , _____)

   Coordinates of the y-intercept (_____ , _____)

   Equation of the axis of symmetry x = _____

3. Click the button [Show graph of $y = $ a x^2] to make the graph of $y = ax^2$ appear.

   Drag the slider for a to change a or click the button [Animate *a*].
   As a changes observe what happens to the graph of $y = ax^2$.
   Use "*sometimes*", "*always*", or "*never*" to complete the following statements about the graph of $y = ax^2$.

   The coordinates of the turning point _____________ change when a changes.
   The coordinates of the x-intercept(s) _____________ change when a changes.
   The coordinates of the y-intercept _____________ change when a changes.
   The equation of the axis of symmetry _____________ changes when a changes.

4. Describe how the graph of $y = ax^2$ changes

   When a is positive and a increases

   ____

   When a is positive and a decreases

   ____

5. When a is more than 1, describe how the graph of $y = ax^2$ is different to the graph of $y = x^2$.

   ____

6. When a is between 0 and 1, describe how the graph of $y = ax^2$ is different to the graph of $y = x^2$.

   ____

7. When a is negative, describe how the graph of $y = ax^2$ is different to what it is when a is positive.

   ____

8. Use "*wider than(not as steep as)*", "*narrower than(steeper than)*" or "*the reflection in the x-axis of*" to complete these statements:

   The graph of $y = 3x^2$ is ______________________ the graph of $y = x^2$.

   The graph of $y = {}^{-}3x^2$ is ______________________ the graph of $y = 3x^2$.

   The graph of $y = \frac{1}{3}x^2$ is ______________________ the graph of $y = x^2$.

   The graph of $y = \frac{1}{2}x^2$ is ______________________ the graph of $y = \frac{1}{4}x^2$.

9. Write an equation for a parabola that "opens up" and is narrower than the parabola with equation $y = x^2$.

   y = ______________

10. Write an equation for a parabola that "opens down" and is wider than the parabola with equation $y = x^2$.

    y = ______________

11. Write an equation for a parabola that has a maximum turning point and is narrower than the parabola with equation $y = x^2$.

    y = ______________

12. Write an equation for a parabola that has a minimum turning point and is wider than the parabola with equation $y = x^2$.

    y = ______________

Click the button [Go to Home Page] to return to the Home Page to begin Activity 2.

Activity 2 – Vertical and Horizontal Translations

2.1 – Vertical Translations

For this activity you will use *Geometer's Sketchpad* to help see what happens to the graph of $y = x^2 + c$ when the number c is changed.

1. In the basic quadratic relationship $y = x^2$ what is the value of c? c = _____
2. In the Home Page of the *Geometer's Sketchpad* file click the button [Vertical Translations] and then click the button [Show graph of $y = \text{x}^\wedge 2 + \text{c}$] to make the graph of $y = x^2 + c$ appear. The graph is a parabola.
   Drag the slider for c to change c or click the button [Animate c].
   As c changes observe what happens to the graph of $y = x^2 + c$.
   Use "*sometimes*", "*always*", or "*never*" to complete the following statements about the parabola with equation $y = x^2 + c$.

   The coordinates of the turning point _____________ change when c changes.

   The coordinates of the x-intercept(s) _____________ change when c changes.

   The coordinates of the y-intercept _____________ change when c changes.

   The equation of the axis of symmetry _____________ changes when c changes.
3. Describe how the graph of $y = x^2 + c$ changes

   when c increases

   ____

   when c decreases

   ____
4. Describe how the graph of $y = x^2 + c$ is related to the graph of $y = x^2$ when c is a positive number.

   ____
5. Describe how the graph of $y = x^2 + c$ is related to the graph of $y = x^2$ when c is a negative number. (In this situation we can also write the equation as $y = x^2 - c$ where <u>c is a positive number</u>.)

   ____
6. Write the equation for a parabola that is the same as the parabola with equation $y = x^2$ but whose turning point is $(0,5)$.

   y = _______________

7. Write the equation for a parabola that is the same as the parabola with equation $y = x^2$ but whose turning point is $(0, {}^{-}7)$.

y = ______________

8. Taking into account Activity 1, write an equation for a parabola that has a maximum turning point $(0, 6)$ and is wider than the parabola with equation $y = x^2$.

y = ______________

9. Taking into account Activity 1, write an equation for a parabola that has a minimum turning point $(0, {}^{-}4)$ and is wider than the parabola with equation $y = x^2$.

y = ______________

10. For the graph of the equation $y = \frac{{}^{-}3}{4}x^2 - 7$

a. Give the following key features of the graph:

Coordinates of the turning point (_____ , _____)

The turning point is a maximum / minimum (Circle the correct answer)

Coordinates of the y-intercept (_____ , _____)

Equation of the axis of symmetry x = _____

b. Complete the following to explain why the graph has no x-intercepts.

*The graph is a parabola. This parabola*______________________________

____

____

____

Therefore, the parabola will never intersect the x-axis.

Click the button [Go to Home Page] to return to the Home Page to begin Activity 2.2

2.2 – Horizontal Translations

For this activity you will use *Geometer's Sketchpad* to help see what happens to the graph of $y=(x-b)^2$ when the number b is changed.

1. In the basic quadratic relationship $y=x^2$ what is the value of b? b = _____
2. In the Home Page of the *Geometer's Sketchpad* file click the button Horizontal Translations and then click the button Show graph of $y=(x\text{-}b)\text{^}2$ to make the graph of $y=(x-b)^2$ appear. The graph is a parabola.
   Drag the slider for b to change b or click the button Animate b .
   As b changes observe what happens to the graph of $y=(x-b)^2$.
   Use "*sometimes*", "*always*", or "*never*" to complete the following statements about the parabola with equation $y=(x-b)^2$.

   The coordinates of the turning point _____________ change when b changes.
   The coordinates of the x-intercept(s) _____________ change when b changes.
   The coordinates of the y-intercept _____________ change when b changes.
   The equation of the axis of symmetry _____________ changes when b changes.

3. Describe how the graph of $y=(x-b)^2$ changes

   when b increases

   ____

   when b decreases

   ____

4. Describe how the graph of $y=(x-b)^2$ is related to the graph of $y=x^2$ when b is a positive number.

   ____

5. Describe how the graph of $y=(x-b)^2$ is related to the graph of $y=x^2$ when b is a negative number. (In this situation we can also write the equation as $y=(x+b)^2$ <u>where b is a positive number</u>.)

   ____

6. Write the equation for a parabola that is the same as the parabola with equation $y = x^2$ but whose turning point is $(8, 0)$.

y = _______________

7. Write the equation for a parabola that is the same as the parabola with equation $y = x^2$ but whose turning point is $(^{-}6, 0)$.

y = _______________

8. Use numbers and the words "up", "down", "right" or "left" to complete the following:

   a. To obtain the graph of $y = (x - 4)^2$ move the graph of $y = x^2$

   _____ units __________

   b. To obtain the graph of $y = (x + 6)^2$ move the graph of $y = x^2$

   _____ units __________

   c. To obtain the graph of $y = x^2 - 9$ move the graph of $y = x^2$

   _____ units __________

   d. To obtain the graph of $y = x^2 + 7$ move the graph of $y = x^2$

   _____ units __________

   e. To obtain the graph of $y = (x - 3)^2 + 5$ move the graph of $y = x^2$

   _____ units __________ and _____ units __________

   f. To obtain the graph of $y = (x + 9)^2 - 3$ move the graph of $y = x^2$

   _____ units __________ and _____ units __________

Click the button [Go to Home Page] to return to the Home Page to begin Activity 3.

Activity 3 – Turning Point Form

In this activity you will use *Geometer's Sketchpad* to see how all the different transformations used to produce the graph of a quadratic equation from the graph of $y = x^2$ are linked in a specific algebraic form of the quadratic equation. This form is called the Turning Point Form and it is given by the equation $y = a(x-b)^2 + c$ where a, b and c are numbers.

1. In the Home Page of the *Geometer's Sketchpad* file click the button Turning Point Form and then click the button Show graph and turning point to make the graph of $y = a(x-b)^2 + c$ appear. The graph is a parabola. The turning point of the parabola is also shown.

   By dragging each slider a, b and c, or by clicking on the Animate buttons for each slider, determine how each of the numbers a, b and c links to one or more of the following transformations of the graph of $y = x^2$:

   - **Vertical translation** - a vertical movement of the parabola
   - **Horizontal translation** - a horizontal movement of the parabola
   - **Reflection in the x-axis**- parabola has a maximum turning point ("opens down") instead of a minimum turning point ("opens up")
   - **Dilation parallel to y-axis by a factor of k** -a narrowing of the parabola (pushed away from x-axis by the factor *k* when $k > 1$) or widening of the parabola (pulled towards the x-axis by the factor k when $k < 1$)

   Summarise what you found by using only a, b, c, "positive", "negative", "between 0 and 1" or "greater than 1" to complete the following for the graph of $y = a(x-b)^2 + c$.

   - The turning point of the parabola is (_____ , _____)
   - The axis of symmetry of the parabola is x = _____ .
   - The turning point of the parabola is a minimum (parabola "opens up") when the number _____ is _______________.
   - The turning point of the parabola is a maximum (parabola "opens down") when the number _____ is _______________.
   - The parabola is wider than the parabola for $y = x^2$ when the absolute value of the number _____ is _______________. The absolute value of the number _____ is the dilation factor.
   - The parabola is narrower than the parabola for $y = x^2$ when the absolute value of the number _____ is _______________. The absolute value of the number _____ is the dilation factor.

- The parabola moves horizontally to the right when the number _____ is _______________ . The number of units it moves is the absolute value of the number _____ .
- The parabola moves horizontally to the left when the number _____ is _______________ . The number of units it moves is the absolute value of the number _____ .
- The parabola moves vertically up when the number _____ is ______________. The number of units it moves is the absolute value of the number _____ .
- The parabola moves vertically down when the number _____ is _______________ . The number of units it moves is the absolute value of the number _____ .

2. The graph for a quadratic equation is obtained by the transformations on the graph for $y = x^2$ listed below:
   - Reflected in the x-axis,
   - dilated parallel to the y-axis by a factor of 8,
   - moved 2 units horizontally to the right and
   - moved 1 unit vertically down.

   a. What is the equation in Turning Point Form? y = ________________

   b. The turning point of the parabola with this equation is (_____ , _____)

3. State what needs to done to the graph of $y = x^2$ to obtain the graph of the equation.

Quadratic Equation	**Reflection in x-axis**	**Dilation by factor of __**	**___ units left/right** (Translation)	**___ units up/down** (Translation)
$y = 3x^2 + 5$				
$y = {}^{-}(x-3)^2$				
$y = {}^{-}5(x+2)^2 - 4$				
$y = \frac{1}{2}(x-3)^2 + 1$				
$y = 4(x+\frac{2}{3})^2 - 6$				

Activity 4 – Parabolic Transformation Creations

(Based on *Graphic Algebra* pages 31-34)

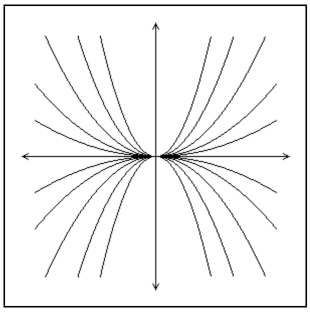
Asp, G., J. Dowsey, K. Stacey and D. Tynan, 2004, *Graphic Algebra*, California: Key Curriculum Press.

For this activity you will use your graphics calculator to create six designs with the graphs of quadratic functions. The designs do not need to be exactly the same as those shown, but they should look similar to those given. After you have created a design record each individual quadratic function rule that you used. Then find and record one rule, two rules or three rules with set brackets { } which, when graphed by your calculator, would create the same design.

1. Fountain reflections

Set the viewing window so that you are looking at the region bounded by:

$$-6 \le x \le 6 \text{ and } -30 \le y \le 30$$



Sometimes when water is spurting out of a fountain its reflection can be seen.

Enter and graph quadratic function rules to draw a fountain of water, and its reflection, like that shown at the right

a. Quadratic function rules used to create the design.

Y1 = ________________________ Y2 = ________________________

Y3 = ________________________ Y4 = ________________________

Y5 = ________________________ Y6 = ________________________

Y7 = ________________________ Y8 = ________________________

Y9 = ________________________ Y0 = ________________________

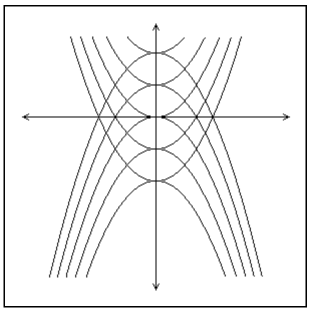
b. One quadratic function rule using { } that would create the same design.

Y1 = ____

2. Fish Kite

Set the viewing window so that you are looking at the region bounded by:

$$-6 \le x \le 6 \text{ and } -20 \le y \le 10$$



Japanese fish kites have streamers flying off the central kite.

Enter and graph quadratic function rules to draw a fish kite like that shown at the right.

a. Quadratic function rules used to create the design.

Y1 = ________________________ Y2 = ________________________

Y3 = ________________________ Y4 = ________________________

Y5 = ________________________ Y6 = ________________________

Y7 = ________________________ Y8 = ________________________

Y9 = ________________________ Y0 = ________________________

b. Two quadratic function rules using { } that would create the same design.

Y1 = ____

Y2 = ____

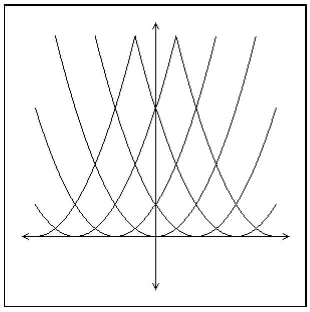
3. Curtain

Set the viewing window so that you are looking at the region bounded by:

$$-6 \le x \le 6 \text{ and } -5 \le y \le 25$$

Theatre curtains often drape in a pattern like that shown at the right.

Enter and graph quadratic function rules to draw a similar pattern.



a. Quadratic function rules used to create the design.

Y1 = ________________________ Y2 = ________________________

Y3 = ________________________ Y4 = ________________________

Y5 = ________________________ Y6 = ________________________

Y7 = ________________________

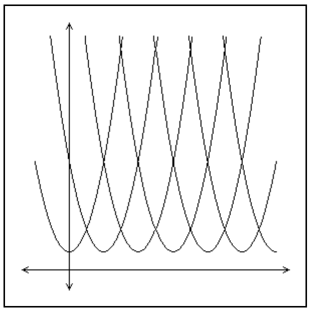
b. One quadratic function rule using { } that would create the same design.

Y1 = ____

4. Parabola Diamonds

Set the viewing window so that you are looking at the region bounded by:

$$-5 \le x \le 30 \text{ and } -2 \le y \le 65$$



Diamonds are found in parabolas! Can you see them?

Enter and graph quadratic function rules to draw a similar pattern.

a. Quadratic function rules used to create the design.

Y1 = ________________________ Y2 = ________________________

Y3 = ________________________ Y4 = ________________________

Y5 = ________________________ Y6 = ________________________

Y7 = ________________________

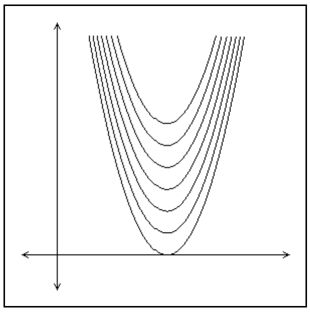
b. One quadratic function rule using { } that would create the same design.

Y1 = ____

5. Running Track

Set the viewing window so that you are looking at the region bounded by:

$$0 \le x \le 20 \text{ and } -5 \le y \le 50$$



The lanes on a running track are not all the same length. Outside runners cover a greater distance than inside runners.

Enter and graph quadratic function rules to draw a similar pattern.

a. Quadratic function rules used to create the design.

Y1 = ________________________ Y2 = ________________________

Y3 = ________________________ Y4 = ________________________

Y5 = ________________________ Y6 = ________________________

b. One quadratic function rule using { } that would create the same design.

Y1 = ____

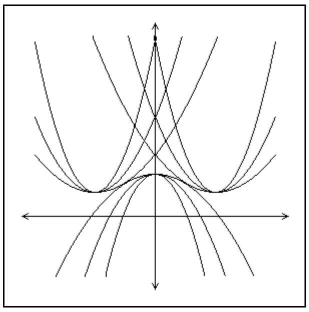
6. <u>No-Name Challenge</u>

Set the viewing window so that you are looking at the region bounded by:

$$-10 \le x \le 10 \text{ and } -10 \le y \le 30$$

This design doesn't have a name.
What name would you give it?

Enter and graph quadratic function rules to draw a similar pattern.



a. Quadratic function rules used to create the design.

Y1 = ________________________ Y2 = ________________________

Y3 = ________________________ Y4 = ________________________

Y5 = ________________________ Y6 = ________________________

Y7 = ________________________ Y8 = ________________________

Y9 = ________________________

b. <u>Three</u> quadratic function rules using { } that would create the same design.

Y1 = ____

Y2 = ____

Y3 = ____