Privileged Pigs Pen Problem

Rosa and Nick are pig farmers who want to build a new pig pen so that they can separate their prize-winning pigs from the other pigs. The new pen is to be made in the shape of a rectangle. It will have a 2-metre wide shallow pool built along one end so that the pigs can splash around in the water. A brick wall will be built around three sides of this pool. For the remaining part of the pen, Nick and Rosa have 28 metres of continuous fencing material to use along the three sides of this part of the pen.

1. Rosa and Nick could make their new pen with these dimensions:

   ![Diagram of the pen with dimensions](image)

   a. Explain why \((10 + 2) \times 8\) gives the total area (including the pool) of this pen.

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   b. What is the total area? ______________________________________________________

   c. What is the total length of the brick wall? _____________________________________

2. The total area of another possible pen that Nick and Rosa could make is given by \((5 + 2) \times 18\).

   a. What is the total area? ______________________________________________________

   b. What are the dimensions of the rectangular part enclosed by the continuous fencing?
      ______________________________________________________________

   c. What is the total length of the brick wall? ________________________________

3. The total area of another possible pen is given by \((___ + 2) \times 12\).

   a. What number belongs in the blank space for this pen? ____________________________

   b. Label the missing dimensions on the diagram below for this pen.

   ![Diagram of the pen with missing dimensions](image)
c. What is the total area? ________________________________

d. What is the total length of the brick wall? ________________________________

4. Consider a possible pen where a side of the rectangular part enclosed by the continuous fencing is \( x \) metres as shown below.

![Diagram of a pen with a side length of \( x \) metres and a wall length of 2m.]

a. Open the file PrivPigPenProb.gsp. Click on the tab Plot of \((x, \text{Total Area})\). Drag point B to see how the point \((x, \text{Total Area})\) moves as \( x \) changes.
   Change \( x \) until you think the point \((x, \text{Total Area})\) is at its highest position.

   Record the values of \( x \) and Total Area for this highest position.

   \[ x = \quad \text{metres} \quad \text{Total Area} = \quad \text{square metres} \]

b. Click the tab Total Area as Function of \( x \) in file PrivPigPenProb.gsp.
   Click on the button Show Locus of Point \((x, \text{Total Area})\).
   Drag point B to see how the point \((x, \text{Total Area})\) moves along this locus.

   Record the following:
   - The smallest value of \( x \) that is possible is close to __________ but it is never equal to this number.
   - The largest value of \( x \) that is possible is close to __________ but it is never equal to this number.
   - Thus, \( x \) is more than __________ but less than ______________

c. For this pen write expressions in terms of \( x \) in the blanks so that

   \[ \text{Total Area} = (\quad + 2) \times (\quad) \]

   gives the total area of the pen in terms of \( x \).

   Choose Plot New Function from the Graph Menu and plot the function rule that you found for the Total Area.

   If the locus of the point \((x, \text{Total Area})\) does not form part of the graph of the function rule, check and redo your work until the locus does form part of the plot of the function rule.
d. Use algebra to find the $x$-intercepts of the graph of the Total Area function rule and use these values to help find the maximum area and the value of $x$ that gives this maximum.

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e. Use algebra to write the rule for the Total Area function in turning point form to find the maximum area and the value of $x$ that gives this maximum.

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5. If Rosa and Nick had 19 metres of fencing instead of 28 metres, work out the new dimensions of the pen of maximum area, and the maximum area.

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6. If Rosa and Nick had $k$ metres of continuous fencing instead of 28 metres, work out the new dimensions of the pen of maximum area, and the maximum area, both as rules in terms of $k$. 

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