# *FENCE THEM IN*

**Aims:** As part of the unit 'Quadratic Functions' students are going to investigate the relationship between area and width of rectangular paddocks for given perimeters.

**At the completion of this investigations students will:**

1. Find the maximum area for a given perimeter of fencing.
2. Use the graphical calculator to generate lists.
3. Create the graphs of Area versus width.
4. Use Nick Solve program to solve two simultaneous equations to determine the values of *a, b* and *c* for the quadratic function $y = ax^2 + bx + c$
5. Use quadratic regression to confirm the found equation.
6. Complete the square to determine the maximum area.
7. Use calculator to confirm maximum area.
8. Investigate the relationships between *a, b* and *c* for the given perimeter.

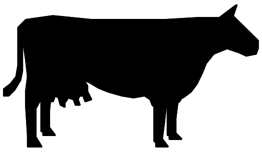
This activity is designed for Year 10 Mathematics as a consolidation of taught concepts in use of quadratic functions. This activity is placed at the end of the quadratic functions unit. The use of the graphical calculator is an integral part in the activity and will involve students using the Statistics function to generate lists and complete quadratic regressions. They will also use the Graphing function of the calculator to confirm the calculated maximum area for a given perimeter.

At this school the TI83 calculator is introduced at Year 9 so the students will have the following prerequisite skills:

- Entering data into SATS list
- Creating graphs from lists
- Entering formula to generate related lists
- Substitution into formula

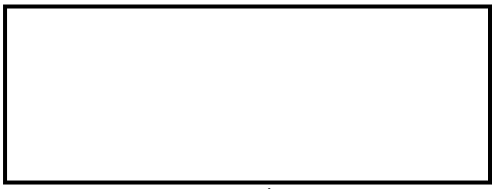
Prior to the commencement of the activity year 10 students will be able to:

- Locate maximum and minimum values
- Recognise the quadratic function $y = ax^2 + bx + c$ general form
- Be aware of simultaneous equations to find unknown pronumerals
- Recognise the quadratic function $y = a(x-h)^2 + k$ for identifying maximum or minimum values
- Complete the square for any quadratic function



**PART 1.** A farmer has just brought some new cattle and needs to fence them in. He has 240 metres of fencing available and wishes to make a 4-sided paddock for the cattle to graze in.

*Width*

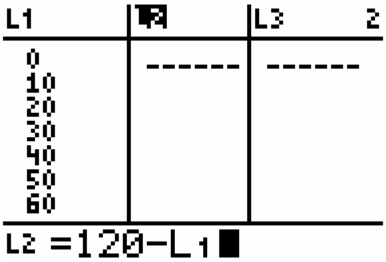


*Length*

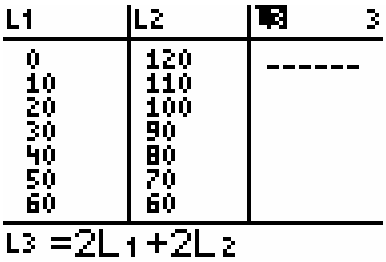
## What size paddock should he make to give his cattle the largest possible grazing area?

Remembering that the perimeter of the paddock must always be 240 metres complete the table and find out which paddock would be best for the farmer's cattle.

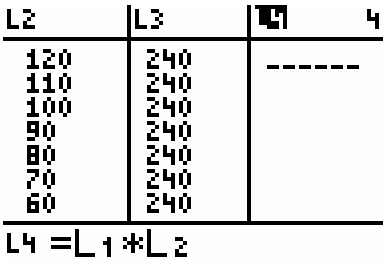
a. Use your graphical calculator to set up the lists required to complete the table. The setups you need to generate the lists for Length, Perimeter and Area for the given values of width are given below. List 1 will need to be extended until $w$ = 120 metres.




The first screen capture shows the set up necessary to generate the width and length for 240 metres of fencing. Explain the setting needed for List 2.




This screen capture shows the set up necessary to generate the width and perimeter for 240 metres of fencing. Explain the setting needed for List 3. Why will it always show 240?



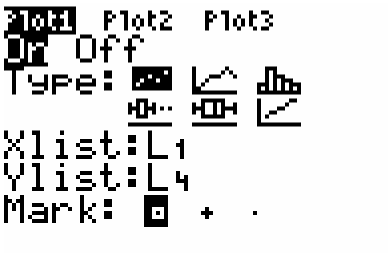

This screen capture shows the set up necessary to generate the Area for the different widths for 240 metres of fencing. Explain the setting needed for List 4.

Copy the list that you have generated using your graphical calculator.

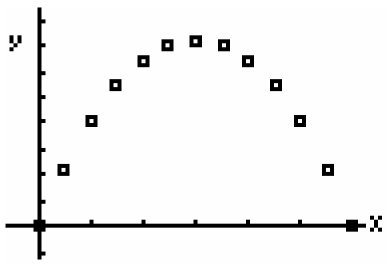
| **Width (m)** | **Length (m)** | **Perimeter (m)** | **Area ($m^2$)** |
|---|---|---|---|
| 0 | 120 | 240 | 0 |
| 10 | 110 | 240 | 1100 |
| 20 | 100 | 240 | 2000 |
| 30 | | | |
| 40 | | | |
| 50 | | | |
| 60 | | | |
| 70 | | | |
| 80 | | | |
| 90 | | | |
| 100 | | | |
| 110 | | | |
| 120 | | | |

You now have all the information you need to investigate the changes in Area as the width is increased from 0 to 120 metres.

b. Sketch the graph of the area versus width using the STAT PLOT function.




9: ZoomStat will automatically scale your graph




c. Give the coordinates for the maximum area. Explain how this point was determined.

Knowing that the function is a quadratic function and follows the rule $y = ax^2 + bx + c$

d. Explain why the value of $c = 0$.

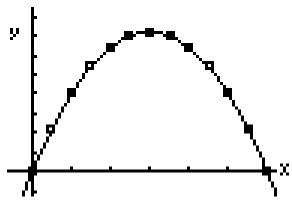
e. Using two points to set up two simultaneous equations find the values of $a$ and $b$.

f. Give the function that will calculate the area ($A$) of the paddock for any given width ($w$).

g. Confirm this function by using the QuadReg in the STAT CALC

**QuadReg L1, L4,Y 1**

Zoom 9:ZoomStat again and you now have the function showing the regression line




h. Your now confirmed function will calculate the area for any width for a paddock

i. Show how this function finds the area for widths of 50m and 100m

j. Use this function to calculate the maximum area. Compare it to the answer you found for question c.

The coordinates of the maximum area can be found by completing the square for the function found in f.

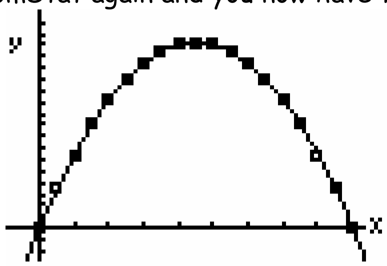
k. By completing the square, confirm that the function in f. can be given as

$$y = -(x-60)^2 + 3600$$

l. What are the dimensions of the paddock for the largest area?

**PART 2:** You are now going to investigate the relationship between the area and width if the perimeter was now 360 metres. Repeat this problem for a perimeter of 360 metres.

a. Use your graphical calculator to set up the lists required to complete another table for the new perimeter. List 1 will need to be extended until *w* = 180 metres. Copy the list that you have generated using your graphical calculator.
b. Sketch the graph of the area versus width using the STAT PLOT function.
c. Give the coordinates for the maximum area. Explain how this point was determined.
d. The graph shows the shape of a quadratic $y = ax^2 + bx + c$. Knowing that c = 0 use two points to set up two simultaneous equations find the values of *a* and *b* but use your program NICKSOLVE to find *a* and *b*..
e. Give the function that will calculate the area (A) of the paddock for any given width (*w*) when the perimeter is 360 metres.
f. Confirm this function by using the QuadReg in the STAT CALC
g. Zoom 9:ZoomStat again and you now have the function showing the regression line.




h. Show how this function finds the area for widths of 50m and 100m
i. Use this function to calculate the maximum area. Compare it to the answer you found for question e.

Again the coordinates of the maximum area can be found by completing the square for the function found in e.

j. By completing the square, confirm that the function in e. can be given as

$$y = -(x-90)^2 + 8100$$

k. What are the dimensions of the paddock for the largest area when the perimeter is 360 metres?
l. The maximum can be found also using the CALC option on your calculator. Use your left and right arrows to bound the maximum area. Find the coordinates of this point. Do they confirm the equation you found when completing the square in j. Explain.

**Questions**

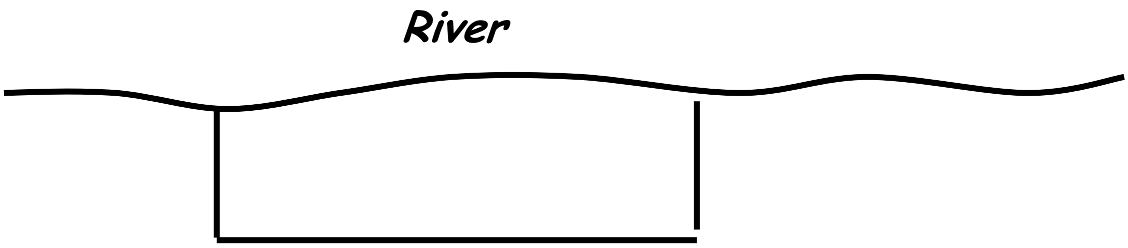
1. What type of paddock has the largest area?

2. Can you find the largest area of paddock with a perimeter of 1000 metres without setting up a table? Explain how.
3. Find the function to calculate the area for 1000m of fencing in the form of $y = ax^2 + bx + c$ ? Explain how you determined the values of *a, b* and *c.*
4. Give the function to calculate this area for 1000m of fencing in the form of $y = a(x-h)^2 + k$? Explain how you determined the values of *a, h* and *k.*
5. Investigate the values of *h* and *k* for the three different perimeters of 240, 360 and 1000 metres. Is there a relationship between these two values and the perimeter of fencing used?
6. Complete the table below

| Perimeter | $y = ax^2 + bx + c$ | Values *a, b, c* | $y = a(x-h)^2 + k$ | Values *a, h, k* |
|---|---|---|---|---|
| 240 metres | | | | |
| 360 metres | | | | |
| 1000 metres | | | | |

7. Can you see a general relationship between the values of *a, b* and *c* and the perimeter? Explain.
8. Can you see a general relationship between the values of *a, h* and *k* and the perimeter? Explain.

**<u>PART 3.</u>** Another farmer has a river running along one side of the paddock so she only needs to fence 3 sides of the paddock to keep her cattle in. She also has 240 metres of fencing to make the 3-sided paddock for the cattle to graze in.




a. Complete another table to determine the maximum area. Explain the set up you used to create your lists.

| **Width (m)** | **Length (m)** | **Perimeter (m)** | **Area ($m^2$)** |
|---|---|---|---|
| 0 | 240 | 240 | 0 |
| 10 | 220 | 240 | 2200 |
| 20 | 200 | 240 | |

b. Find the function to calculate the area (A) for a given width (*w*) in both forms of:

$$y = ax^2 + bx + c$$
$$y = a(x-h)^2 + k$$

c. Show that these two functions confirm the maximum area
d. Again is there a relationship between *a, b* and *c* for $y = ax^2 + bx + c$ ?
e. Again is there a relationship between *a, h* and *k* for $y = a(x-h)^2 + k$ ?
f. Show how you could develop the two rules to calculate area for different perimeters.

**Solutions.**

<u>Part 1:</u>

a. List 2 setting needed is List2 = 120 – List 1 as length of paddock = 120 (half the perimeter) – the width

List 3 setting List 3 = 2*List 1 + 2*list 2 calculates the perimeter of 240 metres which is fixed at 240 metres
List 4 setting List 3 = List 1 * List 2

| L2 | L3 | L4 4 |
|---|---|---|
| 120 | 240 | 0 |
| 110 | 240 | 1100 |
| 100 | 240 | 2000 |
| 90 | 240 | 2700 |
| 80 | 240 | 3200 |
| 70 | 240 | 3500 |
| 60 | 240 | 3600 |

L4(1)=0

List 1 is extended until 120 metres for the width is given

c. Coordinates of the maximum area is (60, 3600)
d. $c$ = 0 as the y intercept for the graph is zero
e. Two simultaneous equations can be formed using any two points from table – for example (20, 2000) and (40,3200)

Gives: For $y = ax^2 + bx + c$
2000 = 400a + 20b
3200 = 1600a + 40b

$a = -1$ and $b = 120$

f. Function to calculate area $A = -w^2 + 120w$
g.

LinReg
y=ax+b
a=-1
b=120
r²=1
r=-1

i. Area when width = 50m
Area $= -(50)^2 + 120(50)$
$= 3500\text{m}^2$

width = 100
Area $= -(100)^2 + 120(100)$
$= 2000\text{m}^2$ as per table of values

j. Maximum area occurs when $w$ = 60m
Area $= -(60)^2 + 120(60)$
$= 3600\text{m}^2$ as per table of values

k. Completing the square

$$\text{Area}(A) = -w^2 + 120w$$
$$= -(w^2 - 120w)$$
$$= -\left[\left(w^2 - 120w + \left(\frac{120}{2}\right)^2 - \left(\frac{120}{2}\right)^2\right]$$
$$= -[w^2 - 120w + 3600 - 3600]$$
$$= -[(w - 60)^2 - 3600]$$
$$= -(w - 60)^2 + 3600$$

l. Shows the maximum for the graph is located at (60,3600) maximum area of 3600m$^2$ when width is 60m.

PART 2:

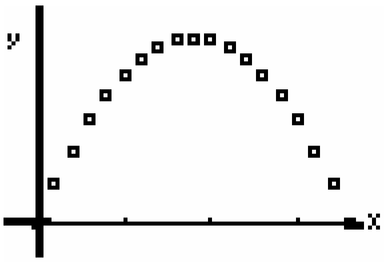
a. For perimeter is 360m

| L1 | L2 | L3 1 |
|---|---|---|
| 0 | 180 | 360 |
| 10 | 170 | 360 |
| 20 | 160 | 360 |
| 30 | 150 | 360 |
| 40 | 140 | 360 |
| 50 | 130 | 360 |
| 60 | 120 | 360 |

L1(1)=0

where list is extended to 180m

b.




c. Coordinates for maximum area occur at (90, 8100) read from table

d. Two simultaneous equations can be formed using any two points from table – for example (20, 3200) and (40, 5600)

Gives: For $y = ax^2 + bx + c$

3200 = 400a + 20b
5600 = 1600a + 40b

$a = -1$ and $b = 180$

e. Function to calculate area $A = -w^2 + 180w$

f.

LinReg
y=ax+b
a=-1
b=180
r²=1
r=-1

h. Area when width = 50m
Area $= -(50)^2 + 180(50)$
$= 6500\text{m}^2$

width = 100
Area $= -(100)^2 + 180(100)$
$= 8000\text{m}^2$ as per table of values

i. Maximum area occurs when $w$ = 90m
Area $= -(90)^2 + 120(90)$

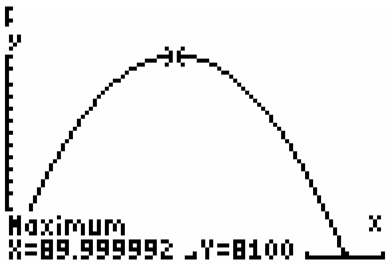
= $8100m^2$ as per table of values

j. Completing the square

$$\begin{aligned}\text{Area}(A) &= -w^2 + 180w \\ &= -(w^2 - 180w) \\ &= -\left[\left(w^2 - 180w + \left(\frac{180}{2}\right)^2 - \left(\frac{180}{2}\right)^2\right]\right. \\ &= -[w^2 - 180w + 8100 - 8100] \\ &= -[(w-90)^2 - 8100] \\ &= -(w-90)^2 + 8100\end{aligned}$$

k. Shows the maximum for the graph is located at (90,8100) maximum area of $8100m^2$ when width is 90m

l. Using $Y1 = -X^2 + 180X$




Questions:

1. What type of paddock has the largest area? Paddocks that are square
2. Can you find the largest area of paddock with a perimeter of 1000 metres without setting up a table? Explain how. The paddock with the largest area would have:

length = width = $\frac{1}{4}$ perimeter

Length = $\frac{1}{4}$ of 1000

Length = 250 m

Area = 250 x 250

= $62500m^2$

3. Find the function to calculate the area for 1000m of fencing in the form of $y = ax^2 + bx + c$ ?

$$y = -x^2 + 500x$$

4. Give the function to calculate this area for 1000m of fencing in the form of $y = a(x-h)^2 + k$?

$$y = -(x-250)^2 + 62500$$

Explain how you determined the values of *a, h* and *k*. a = –1 *h* = perimeter/4 and *k* = $(\text{perimeter}/4)^2$ or $250^2$

5. Investigate the values of *h* and *k* for the three different perimeters of 240, 360 and 1000 metres. Is there a relationship between these two values and the perimeter of fencing used?

6. Complete the table below

| Perimeter | $y = ax^2 + bx + c$ | Values *a, b, c* | $y = a(x-h)^2 + k$ | Values *a, h, k* |
|---|---|---|---|---|
| 240 m | $A = -w^2 + 120w$ | *a* is always –1<br><br>*b* is P/2<br><br>*c* always = 0 | $y = -(x-60)^2 + 3600$ | *a* is always –1<br><br>*h* is P/4<br><br>*k* always = $(p/4)^2$ |
| 360 m | $A = -w^2 + 180w$ | | $y = -(x-90)^2 + 8100$ | |
| 1000 m | $A = -w^2 + 500w$ | | $y = -(x-250)^2 + 62500$ | |

7. Can you see a general relationship between the values of *a, b* and *c* and the perimeter? Explain. As shown in the table above
8. Can you see a general relationship between the values of *a, h* and *k* and the perimeter? Explain. As shown in the table above

PART 3: Three sided paddock

a.

| L1 | L2 | L3 1 |
|---|---|---|
| 0 | 240 | 240 |
| 10 | 220 | 240 |
| 20 | 200 | 240 |
| 30 | 180 | 240 |
| 40 | 160 | 240 |
| 50 | 140 | 240 |
| 60 | 120 | 240 |

L1 ={0,10,20,30,…

List 2 = 240 – 2xList1; List 3 = 2xList 1 + List 2
List 4 = List 1 x List 2

b. Find the function to calculate the area (A) for a given width (*w*) in both forms of:

$$y = -2x^2 + 240x$$

$$y = -2(x-60)^2 + 7200$$

c. Show that these two functions confirm the maximum area

When $x = 60$

Area = $-2(60)^2 + 240 \times 60$ Area = $-2(60-60)^2 + 7200$

Area = 7200m$^2$ Area = 7200m$^2$

d. Again is there a relationship between *a, b* and *c* for $y = ax^2 + bx + c$ ?

In this form $a = -2$ *b* = perimeter and $c = 0$

e. Again is there a relationship between *a, h* and *k* for $y = a(x-h)^2 + k$ ?

In this form $a = -2$ *h* = perimeter/4 and *k* =twice(perimeter/4)$^2$

f. Show how you could develop the two rules to calculate area for different perimeters. As shown in question e