Marina's Fish Shop Student Worksheet Name_____

This sign

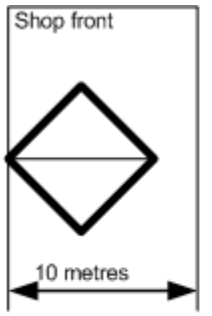
OR

this sign

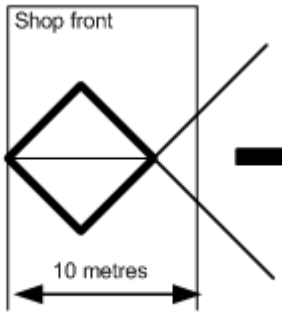
OR

this sign?

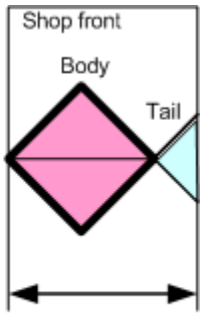
Marina owns a fish shop, and wants to create a new sign above the shop. She likes geometric ideas, and thinks a square with a triangle looks like a fish. Marina draws a square with a horizontal diagonal, starting from the left wall of her shop. This makes the body. Then she extends two sides of the square as far as the right wall of the shop. This makes the tail. The shop is 10 metres wide. Marina soon realises that there is more than one possible configuration (see above) and wonders, "What is the best possible sign?" She uses mathematics to investigate.



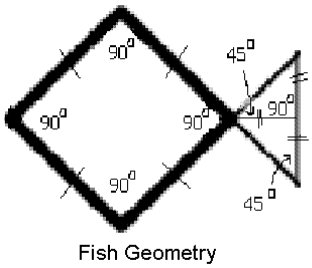
Draw a square with horizontal diagonal



Extend the sides



Make a triangle with end of shop



Fish Geometry

Activity 1: OBSERVING VARIATION (Use page 1.1 of Nspire file 'FishShop_Handheld.tns')
Use the flexible fish diagram to create many different fishes for Marina and observe them closely.

1. Write down 3 things that are the same about all the fishes made by the flexible diagram.

_____ _____ _____

2. Write down 3 ways in which the fishes made by the flexible diagram differ.

_____ _____ _____

3. The total area of the fish appears to change as the body length of the fish changes. Describe in one sentence what appears to happen to the total area of the fish as you increase the length of the body of the fish from 0 m to 10 m.

4. On your flexible fish diagram the variable *b* represents the length of the _____ of the fish measured in _____ (units).

5. Manipulate your flexible fish diagram to find visually the approximate value of *b* that:

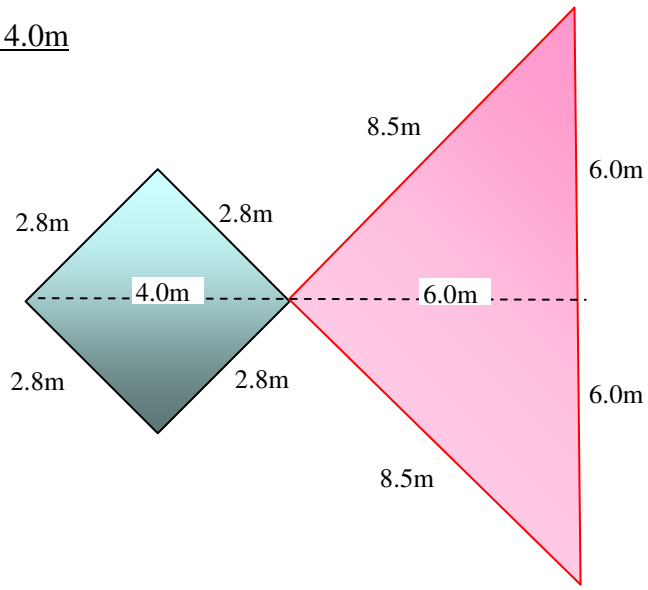
(a) makes the fish which appears to have the largest possible area: b = _____
(b) makes the fish which appears to have the smallest possible area: b = _____

Activity 2: CALCULATING TOTAL AREA

Use this space for all written calculations
(and Calculator page 1.2 to assist you if you wish)

A small-bodied fish

Body length is 4.0m



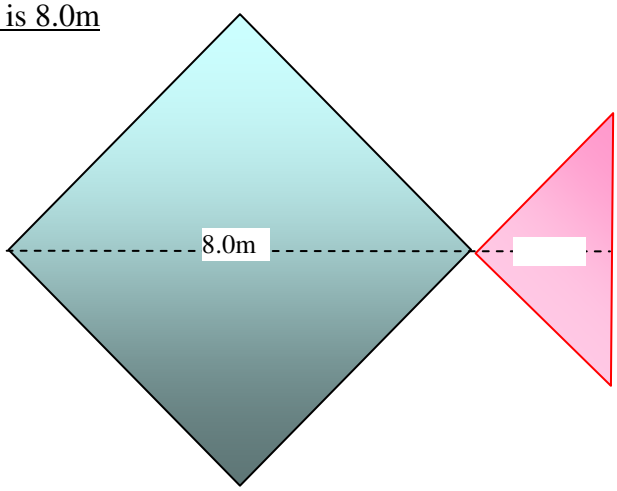
6. The area of the body is _____m².

7. The area of the tail is _____m².

8. Therefore the total area of this small-bodied fish is _____m².

A large-bodied fish (Remember: the fish is still 10m long)

Body length is 8.0m



9. The length of the tail is now _____m.

10. Label the other parts of the tail that have this same measure.

11. Label each side of the square as being L metres long.
Use Pythagoras' theorem to form and solve an equation to find L.

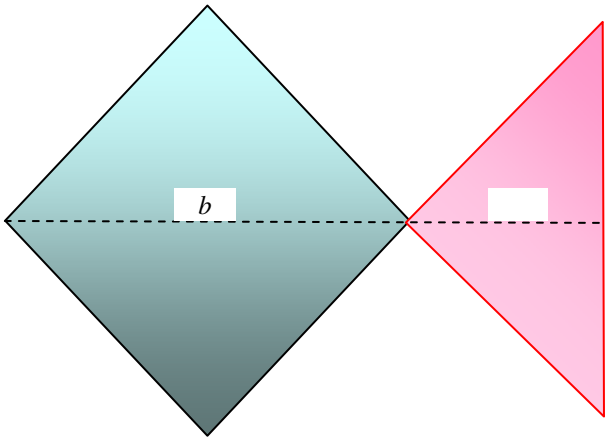
12. L = _____m (1 decimal place)

13. For this large-bodied fish:

The area of the body is _____m², the area of the tail is _____m², and the total area of this large-bodied fish is _____m².

Any 10m fish

Body length is *b* m



14. Use the previous numerical examples as a guide to find an algebraic expression for the total area of any such fish in terms of *b*.

Hints: To start, find the length of the tail;
Next, find and label other necessary sides;
Use the same rules and formulae as before.

Area of the body: _____m²,

Area of the tail: _____m²,

The total area of any 10m fish is

_____m²

Activity 3: GRAPHING THE AREA FUNCTION FROM DATA (Use p1.3, 1.4, 1.5 of Nspire file)

Your result for the last part of Activity 2 is a rule linking the total fish area (let's call it *Total_Area*) to the body length (which we've called *b*). We found in both Activities 1 and 2 that by changing the value of *b* we get a changed value of *Total_Area*, so we say that *Total_Area* is a function of *b*.

15. Write the function rule (using *b*) in this space: *Total_Area* =_____.

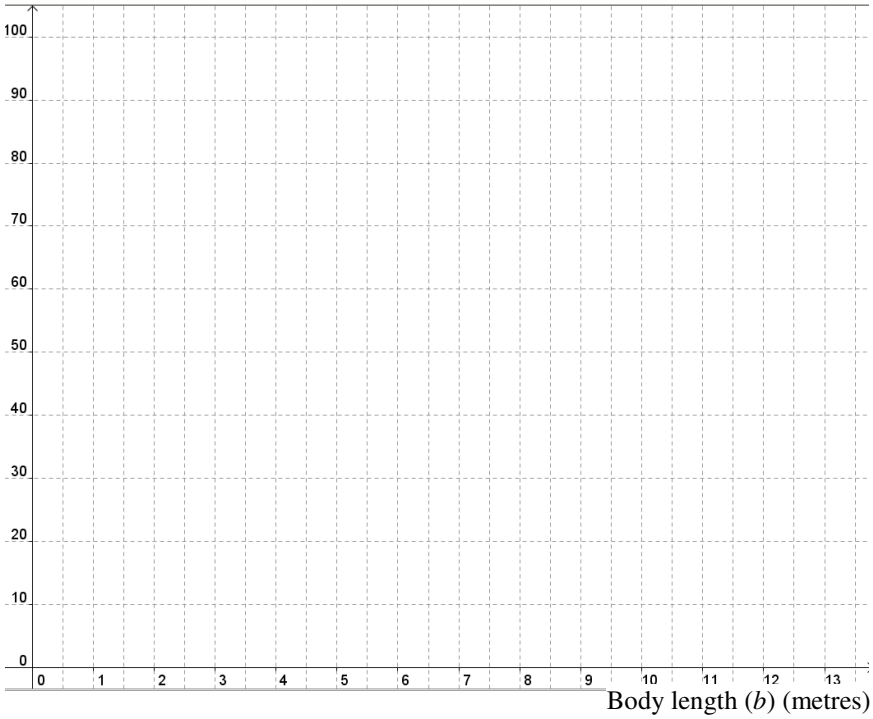
16. How do you know this function is quadratic?

17. By using written calculations or your Nspire page 1.2, complete the following table:

Calculations here:

***b* m**	***Total_Area* m²**	**Coordinates (*b*, *Total_Area*)**
4	**44**	
8	**36**	
1.5		

18. You now have the coordinates of three points; plot them on the given set of axes below. Label the vertical axis appropriately.



19. Using a pencil, connect your three points with an appropriate graph. Describe in words what the graph tells you about the area of the fish.

20. Use the graph to provide new estimates for the largest and smallest fish:

- When *Total_Area* is the largest, what is the value of *b*? _____
- When *Total_Area* is the smallest, what is the value of *b*? _____

21. Use the data capture technique to collect values from the flexible fish diagram (Nspire p1.3) for 10 different values of *b*. The values for *b* will appear in the 1st column of the spreadsheet (Nspire p1.4), and the values for *Total_Area* will appear in the second column. The data will be shown as points on the scatterplot (Nspire p1.5). *Note: be sure to capture points for a large variety of body sizes.*

22. Determine the graph that best goes through the given points. To do this, your Nspire uses a process called *regression*. To do this from the scatterplot screen (page 1.5) follow these steps:

- press **MENU** (menu)
- then (4) **4:Analyze**
- then (6) **6:Regression**
- then (4) **4:Show Quadratic.**

Notice that the rule for the graph uses the standard *x* and *y* format.

23. Use a pen to redraw the graph in step 19 more accurately.

24. Write the rule as shown on your calculator screen: _____

25. Rewrite this rule using the variables *Total_Area* and *b*: _____

26. Is this the same as the rule used in Step 15? _____Show working out to verify your answer.

Activity 4: FINDING THE MINIMUM AREA FROM THE GRAPH (Use page 1.6 of Nspire file)

At night, Marina's sign will show the interior of the fish design lit up against a black background. Marina is both an environmentalist and a smart businessperson, and wishes to use as little area of lighting as possible, to save money and energy. This activity finds the measurements of the sign with minimum area.

27. What name was given above to the variable that Marina wants to minimise? _____

28. Use Nspire to draw the graph of the quadratic function for the total area of the fish. Nspire requires you to use the pronumeral *x* (rather than *b*) for the body length, and $f_2(x)$ for the total area.

29. Attach a point that moves along the parabola. To do this, access the "Point On" tool using the following steps:

- Press **MENU** (menu)
- then (6) **6: Points & Lines**
- then (2) **2: Point On**. Click on your parabola, to attach the point.
- Press **Escape** (esc) to exit that menu.

Grab the point to move it up and down the parabola. You will notice the coordinates of the point changing as you do this. Write down what these coordinates tell you about the area of the fish.

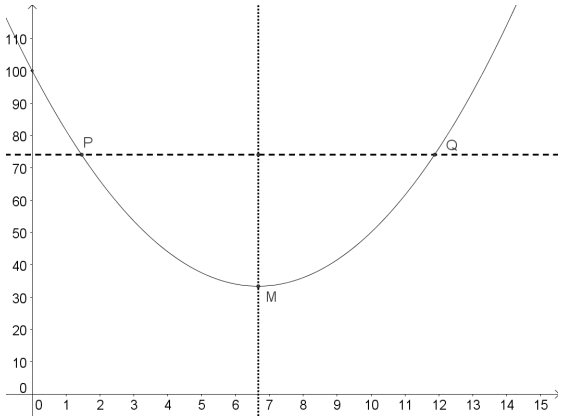
30. Recall that Marina wants the fish sign with the smallest possible area. Relocate your movable point to the position where this happens.
Write the coordinates of this point: (,).
How does Nspire indicate the minimum point?_____
Now plot this point on the set of axes used in Activity 3, step 18.

31. Sketch a labeled diagram to show Marina the measurements for her sign.

Activity 5: FINDING THE MINIMUM AREA EXACTLY (Use pages 1.2 and 1.6 of Nspire file)
Marina has recently found out that she is a descendant of Pythagoras; and there's a family reunion this summer on the Greek island of Samos. She knows that her relatives are interested in her business and she also knows that when she explains how she's designed her fish sign, they will want an exact value for b.

Recall that the turning point of a parabola can sometimes be found easily using the *x*-intercepts and symmetry of the graph; however, there are no *x*-intercepts for this graph, so you need a different tactic.

32. On Nspire page 1.6, sketch a horizontal line of your choice which intersects the parabola twice. This will be in the form $f3(x)$ = some number between 50 and 90. See diagram below.



[*Hint*: if using Nspire, solve the equation $f2(x) = f3(x)$ for *x*, using **MENU** (menu) **then** (3) **3: Algebra, then** (1) **1: Solve** and using the syntax Solve(*equation*, *x*)]

The points P and Q are the intercepts. They lie on this line, and also on the parabola.
Write down an equation which can be solved to give the *exact x*-values for P and Q.

Solve this equation using pen-and-paper or Nspire (p1.2)

Write the two solutions below:

x = and *x* =

33. Use your knowledge of symmetry and algebra to find the *exact* coordinates of the turning point M. Show appropriate working below.

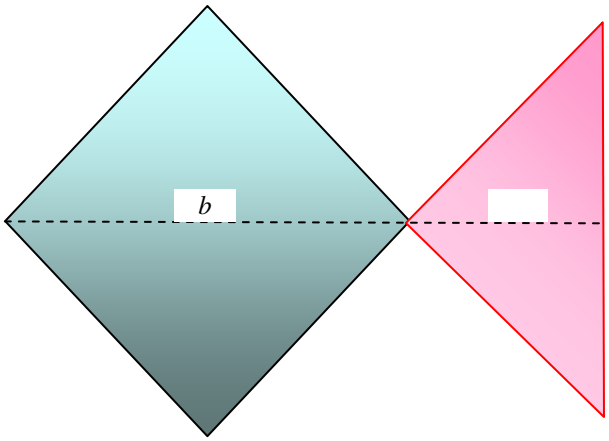
[*Hint*: Your two values (in exact surd form) are the *x*-coordinates of the points P and Q. Notice how the turning point is in the "middle" of these, by symmetry. Use this idea and page 1.2 to calculate the *exact x*-coordinate of the turning point. The exact *y*-coordinate can be found by replacing the pronumeral *x* in $f2(x)$ with the *exact x*-coordinate found above.]

34. Now state the exact minimum total area and the exact body length when the minimum occurs.

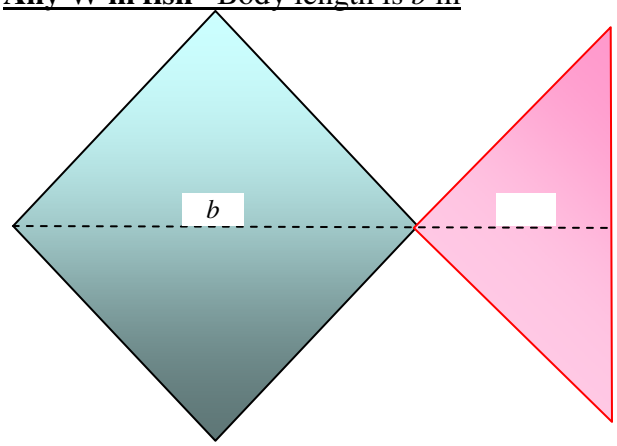
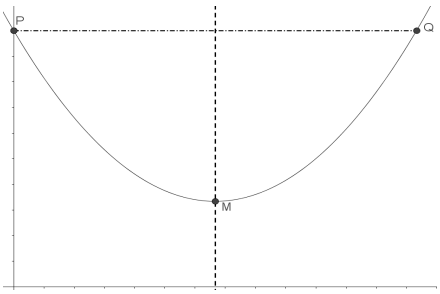
Activity 6: CHALLENGE: PRODUCING A GENERAL SOLUTION

Why is the number 100 prominent in the minimum total area? In this activity, you will find out how to predict the minimum total area and the related body length from the width of the sign. There are two approaches. In step 35 you repeat the calculations done above with another length and guess the relationship from this data. In step 36, you do the calculations with algebra, to get a proof. Choose to do one or the other.

35. This is a numerical approach. Find the result for a 14m fish, and look for a pattern.

Any 14m fish Body length is *b* m  	Apply appropriate formulae as in Activity 2. New tail length = _____ m Area of the body = _____ m² Area of the tail = _____m² Total area of 14m fish = _____ m² Use Activity 5 procedures to find the exact minimum area for this new fish sign. Min. area = _____ m² when b = _____m Guess the minimum area for a fish sign of total length W and the related body length. Min. area = _____ m² when b = ____ m

36. A symbolic approach – representing the width of the sign by W metres.

Any W m fish Body length is *b* m  	As in Activity 2, represent the sides of the square as L, the new tail length as _____, and apply appropriate formulae as before. Area of the body: _____m², Area of the tail: _____m², The total area of any W m fish is _____m²
 	To use the ideas of Activity 5, find the coordinates of P and Q in the special case when P = (0, W²). This special value of P makes the algebra easy. Find the coordinates of Q *algebraically*, by solving the equation [expression for area = W²] Q = (____ __, W²). Find the coordinates of M, using symmetry. M = (_____, _____) Describe the fish with minimum area.