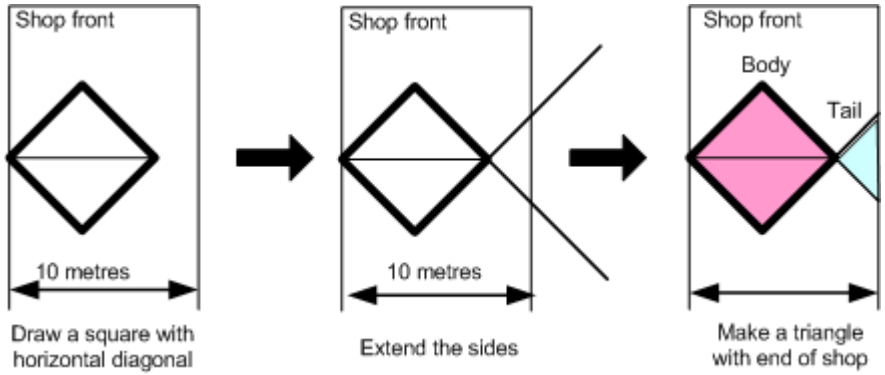
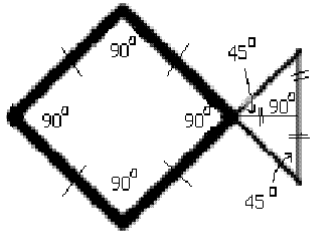
LESSON PLAN -Marina's Fish Shop

Applying quadratic functions using Nspire CAS (Year 10)

Marina owns a fish shop, and wants to create a new sign above the shop. She likes geometric ideas, and thinks a square with a triangle looks like a fish. Marina draws a square with a horizontal diagonal, starting from the left wall of her shop. This makes the body. Then she extends two sides of the square as far as the right wall of the shop. This makes the tail. The shop is 10 metres wide. Marina soon realises that there is more than one possible configuration (see above) and wonders, "What is the best possible sign?" She uses mathematics to investigate.





Fish Geometry

Marina is aware that certain aspects of both the body and the tail of the fish seem to change with each configuration, and seeks to develop some methods for recording and analysing these changes. Students will use both written and technology-assisted mathematics in Activities 1-3 to explore the patterns of data created by these configurations.

At night, Marina's sign will show the interior of the fish design lit up against a black background. She is both an environmentalist and a smart businessperson, and wishes to use as little area of lighting as possible, to save money and energy. In Activities 4 and 5, students will find the measurements that Marina's sign should have to ensure this happens. In Activity 6 the results are generalised.

Grade level: Year 10

Activity time: 100 minutes

Aims: The students will

- Explore data generated through a problem involving area
- Use multiple representations of that data to create a mathematical (quadratic) model
- Apply algebraic and geometric knowledge to express the model symbolically
- Use regression to confirm the model
- Solve the problem
- Generalise the solution
- Reflect on the process of creating mathematics from a problem

Assumed concept knowledge: Area of a square and triangle, Pythagoras' theorem, simple transposition of formulae

Assumed Nspire knowledge (students):

- Downloading Nspire files from teacher handheld device
- Moving between pages of an Nspire multi-page document
- *Calculator application*: Solve command; exact/approximate mode settings
- *Graphs&Geometry application*: "grabbing" a point and moving it using NavPad; entering a function; using "Point On" to trace along a graph; altering window settings

Additional assumed Nspire knowledge (teachers):

- Downloading an Nspire file to student handheld devices
- Using Nspire CAS computer software with data projector

Summary of Lessons: Students are presented with a scenario involving the design of a commercial sign with mathematical (geometric) constraints. They approach their analysis using the following systematic approach:

Activity Number	Activity Name	Overview of tasks
1	**OBSERVING VARIATION**	Visual estimation of effect of body length change on total area change is done through the use of a dynamic geometrical diagram
2	**CALCULATING TOTAL AREA**	Total area is calculated for each of two specific possible configurations; an algebraic model for any such configuration is then developed.
3	**GRAPHING THE AREA FUNCTION FROM DATA**	Data generated from technology-assisted measurements are listed in a spreadsheet and graphed on a scatterplot, both being linked to a dynamic geometric diagram. Consistency of data with algebraic model is confirmed.
4	**FINDING THE MINIMUM AREA FROM THE GRAPH**	Motivation for finding minimum area given in problem context. Technology-assisted analysis of quadratic function graph yields approximate result.
5	**FINDING THE MINIMUM AREA EXACTLY**	Algebraic and graphical techniques are used to find exact values for the minimum area and its corresponding body length.
6	**CHALLENGE: PRODUCING A GENERAL SOLUTION**	A general solution, applicable to a sign of any given length, is found. Numerical and symbolic approaches are explored. May be seen as an extension activity.

Technical notes: The diagrams used on Nspire will already be drawn, and the spreadsheet and scatterplot will have been pre-labeled with key formulae entered.

The following files will be required before the lesson commences:

- FishShop_Handheld.tns Staff can download this file to their own handhelds using Nspire Computer Link software, and then transfer it to relevant students' handhelds. It is be assumed that Nspire CAS version 1.4.11643 from 2008-Jul-09 (Home, then 8:System Info, then 4:About) is in use. NOTE: It is ideal if teachers saved the downloaded file twice, with one version being for students' use and one for teachers to experiment with ahead of time. Enough time should also be allowed for all students to have the file ready to use before the first lesson starts.

- FishShop_Teachers.tns This Nspire CAS computer software file is the one staff will use in class to teach the lesson. It is assumed that Nspire CAS version 1.4.11654 is in use. It will contain all the screens seen in the student file, and with a layout better suited for a classroom demonstration. (This occurs in Page 3, which is a triple-pane window with pages 1.3, 1.4 and 1.5 of the student file.) When using this file in the lesson, the intended view is "Normal", maximized with "slides" appearing down the left side of the screen. Note that this is neither the "Presentation View" nor the "Handheld View" (which should not be used as diagrams become distorted). Again, teachers are well-advised to have a working familiarity with this file, saving an additional copy for that purpose.

Guide to Contents of Nspire files

Student file (Handheld) Page No.	Teacher file (Computer) Page No.	Application/Contents
1.1	1	"No Measurements" flexible fish diagram (G&G)
1.2	2	Calculations page (Calculator)
1.3	3	"With measurements" flexible fish diagram
1.4		Spreadsheet for data captured in "With Measurements" page (L&S)
1.5		Scatterplot for spreadsheet data (D&S)
1.6	4	Coordinate axes for function graphing (G&G)

Assessment and evaluation: Students could be asked to detail in their own words what they have found. Teachers could assess the mathematical reasoning and terminology used in any responses submitted for this task. Alternatively, under test conditions students could attempt a solution to a similar problem where the overall horizontal space containing the sign is say, 8 metres wide, submitting their altered Nspire file with answers to questions shown on a Notes page.

Further Extensions: Able students may be interested to develop the appropriate mathematical model for a similar situation with a different configuration, such as a triangle-square-triangle design. A second problem, whereby Marina seeks to make the body and tail areas equal (through the motivation of visual "balance" in the sign being better for increased sales) may be posed. Alternatively, the *perimeter* of the fish shape might be explored (leading to a linear, rather than quadratic, relationship which might be good at the Year 9 level). Others, including teachers, may be interested in using the Hide/Show features of their file to explore the construction methods used to create the original dynamic diagram.

Technology requirements: Student Nspire handhelds to have file FishShop_Handheld.tns downloaded prior to lessons. (Start lesson with device off, however). Teacher requires a data projector and a laptop with 2 files on its desktop:

FishShop_Teacher.tns [Nspire CAS computer file]
FishShop_StdntWksht.doc (Word file of student worksheet).

Other materials: Each student must have a pen/pencil and "Marina's Fish Shop" student worksheet. Teacher needs whiteboard markers for informal data recording/comments/clarification. 3 to 5 Overhead projector acetate sheets (photocopied with graph grid of step 18, p4 of worksheet) and OHP pens are also required.

Time	Nspire CAS computer file page reference	Teacher activity	Student activity	Technology tips
10 min	Page 1 "No Measurements"	*Note: At the start of this activity, students should have their Nspire turned off, and they should have been given the worksheet – they are required to focus on the teacher's presentation.* **1. OBSERVING VARIATION** • Project Page 1 of student worksheet on screen; briefly summarise the situation. • Diagram examination: clarification of Marina's 3-stage construction process and geometric properties; define key terms *body*, *body length, tail*; constancy of 10m overall length. • Project Nspire file; demonstrate design variability by manipulating flexible fish diagram on "No Measurements" page 1. • Instruct students to complete Steps 1-4 of activity 1. • Discuss answers. • Instruct students to open their Nspire file and to manipulate their diagram on page 1.1. • Instruct students to complete step 5; visual estimation is all that's required.	Read p1 of Student Worksheet Complete Steps 1-4 of Activity 1 Open Nspire file and manipulate fish diagram "No measurements" p1.1. Complete step 5 of Activity 1.	Maximise screen on p1 (and p3) to prevent distortion of diagrams. Point at right end of black segment must blink (not segment itself); **ctrl** **click** to "grab" before moving Students to be advised to "tap" NavPad when moving left and right, not hold it down.

Time	Nspire CAS computer file page reference	Teacher activity	Student activity	Technology tips
15 minutes (10 min for student independent work then 5 min teacher-directed completion of generalised statement (Step 14)	Page 2 Calculator application	**2. CALCULATING TOTAL AREA** • Refer students to worksheet Activity 2, p2-3. Direct students to note progression from fully-measured to minimally-measured to generalised diagrams. • Remind students of constancy of 10m overall length as well as unchanging geometric relationships in all 3 diagrams. • Instruct students to complete Activity 2, showing all necessary working out and using their Nspire Calculator application p1.2 for calculations (and/or algebra). • Provide assistance as required including reminders of area formulae, units required, rounding protocols. • After 10 minutes, give detailed instruction for completion of "Any 10m fish" algebraic modeling of generalised case. • Instruct students to leave the generalised result in $\frac{1}{2}b^2+(10-b)^2$ format. • Discuss findings as time allows	Worksheet Activity 2 (Area calculations). Students to use Calculator application (Nspire page 1.2) for this basic arithmetic (and/or algebra)	Fraction template (for ½) found using **ctrl** (ctrl) **catalogue** (esc) If using Nspire for algebra, use of Solve($L^2+L^2=8^2, L$) syntax; then **ctrl** (ctrl) **Enter** (enter) for decimal approximation

Time	Nspire CAS computer file page reference	Teacher activity	Student activity	Technology tips
25 minutes	Nspire CAS computer page 3 Triple-pane (Fish diagram with measurements, spreadsheet, scatterplot)	**3. GRAPHING THE AREA FUNCTION FROM DATA** • Remind students that graphing is a visual way of showing how the area varies with the body length. • Introduce students to worksheet Activity 3, p3-5, noting that there are calculations to perform (again on their Nspire p1.2), spaces to fill in with numbers, expressions and sentences, and a graph on which to plot points and sketch graphs (suggest pencil?). • Instruct them to complete steps 15 through 20; provide assistance as required. • Hand OHP sheet and pen to each of 3 to 5 selected students; instruct them to plot their points and sketch from Step 18,19; project their results; compare and contrast. • After a few (8?) minutes, demonstrate data capture procedure on Nspire p3; remind students they have these 3 screens separately on their pages 1.3, 1.4 and 1.5. Two or three points only. Note linking of pages; discuss labels on spreadsheet columns and scatterplot axes. • Instruct students to complete steps 21 through 26; provide assistance as required. • Discuss findings as time allows.	Note structure and varied answer formats for Activity 3. Complete Worksheet Activity 3, steps 15 through 20. Use Nspire page 1.2 for all calculations. 3 to 5 students share answers with class via OHP sheets. Learn data capture technique from teacher demo. Complete steps 21 through 26; use Nspire pages 1.3, 1.4 and 1.5.	Data capture technique: Start on Nspire p3 using flexible fish diagram "With measurements" pane activated. Press **ctrl** (ctrl) **decimal point** (.); "grab" right endpoint of black segment to alter diagram; press **ctrl** (ctrl) **decimal point** (.); repeat for third point if required. Note: Computer version requires cursor in actual page 3 diagram pane, not in "slide" of that on left side of page. Regression technique: **MENU** (menu) then (4) **4:Analyze** then (6) **6:Regression** then (4) **4:Show Quadratic.**

Time	Nspire CAS computer file page reference	Teacher activity	Student activity	Technology tips
25 minutes	Nspire CAS computer page 4	*If required, summarise findings from previous lesson under the banners "Observing variation", Calculating total area" and "Graphing the area function from data". Discuss any queries regarding Activities 1-3 from worksheet.* **4. FINDING THE MINIMUM AREA FROM THE GRAPH** • Read problem posed at start to worksheet activity 4, page 5. • Open Nspire Graphs&Geometry page 4 (handheld page 1.6 for students) and note its structure, the labeled axes, the scale etc – compare to scatterplot used in activity 3. Remind students of the "fn(x) =" structure of any functions entered on this page. • Instruct students to complete activity 4; provide assistance as required. Stress the task of finding the minimum fish area. • Note how this method provides an approximate solution [minimum area 33.33m²]; some may infer its exact value. • Discuss details of diagram (step 31)	Note structure and varied answer formats for Activity 4. Complete Worksheet Activity 4, steps 27 through 31. Use Nspire page 1.6 for graphing.	Note The function students will be graphing here is $f2(x)$; the function $f1(x)$ has already been used (and its name hidden) previously in the file to construct the flexible fish diagram. Point On technique: **MENU** (menu) then (6) **6: Points & Lines** then (2) **2: Point On.**

Time	Nspire CAS computer file page reference	Teacher activity	Student activity	Technology tips
25 minutes	Nspire CAS computer pages 2 and 4	**5. FINDING THE MINIMUM AREA EXACTLY** • Read problem posed at start to worksheet activity 5, page 6, and discuss intended method: solving the simultaneous linear/quadratic, average of exact solutions, substitution to find exact minimum area – using handheld pages 1.6, 1.2. • Instruct independent students to complete the activity. • Refer other students to hints in steps 32 and 33. • Using whiteboard, record student x-coordinates of P and Q (will vary depending on $f3(x)$ value). Note common structure (assuming $f3(x)=t$) to be $\frac{20\pm\sqrt{6t-200}}{3}$, helping to "predict" average of $\frac{20}{3}$. • Verify minimum area as $\frac{100}{3}$ m².	Complete Worksheet Activity 5. Use Nspire page 1.6 for graphing, then page 1.2 for equation solving, calculation, and substitution..	Illustrated example in step 32 done on Nspire as follows: Page 1.6: With cursor in entry line across from $f3(x)=$, key in 74 and press **ENTER** enter. Find intersection points on Nspire as follows: Page 1.2 **MENU** menu then 3 **3:Algebra** then 1 **1:Solve** then syntax Solve($f2(x)=f3(x),x$) then **ENTER** enter**.** [$x=\frac{20\pm2\sqrt{61}}{3}$] Find average to be $\frac{20}{3}$, and minimum area using $f2(\frac{20}{3})$

Time	Nspire CAS computer file page reference	Teacher activity	Student activity	Technology tips
30 minutes	Nspire CAS computer pages 2 and 4	*Depending on time this might be set entirely as an out-of-class activity for more capable students. It may also be started in class with all students, using varying degrees of teacher input as warranted. Two approaches are suggested; either or both may be explored.* **<u>6. CHALLENGE: PRODUCING A GENERAL SOLUTION</u>** • Use the rather simple exact minimum area being $\frac{100}{3}$ m² as a lead into step 35, page 7. • Give students overview of the numerical and symbolic approaches. Mention they may need to complete written work on back of page. • Note that numerical approach will involve a repetition of Activity 4 and 5 procedures (with 14 replacing 10) and will thus involve Nspire; generalization then follows. • Note that symbolic approach is best taken using pen-and-paper methods only. • Instruct students to either continue with activity 6 or suggest extension activities (see page 4 of this document) • Discuss generalisations which arise.	Complete Worksheet Activity 6. Complete extensions as indicated by teacher	As per activity 4,5