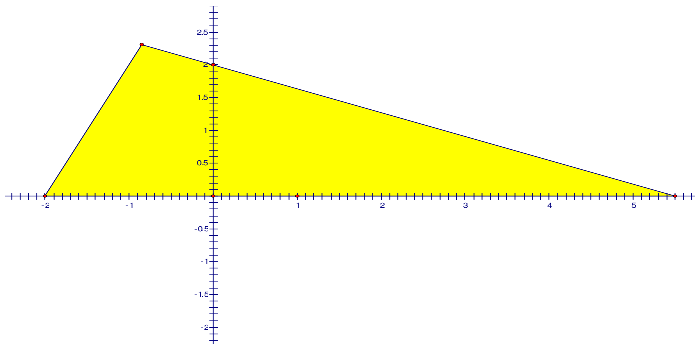
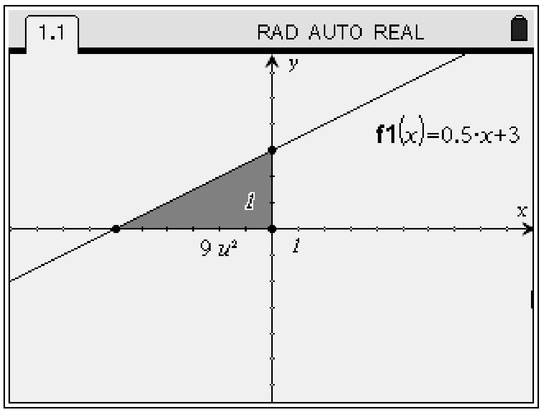
# <u>Linking Linear Functions and Measurement: Investigating using CAS</u>

(Nspire version)



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The University of Melbourne
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# Contents






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Background to the Unit of Work

| | |
|---|---|
| Year Level | 10 (with 1+ years' exposure to CAS with varying degrees of use) |
| Time allocation | 6 lessons, comprising normal class meeting times in one 10-day cycle with 4 periods (assume 80 minutes each) per day. |
| Assumed Knowledge | • Sketch graphs (by-hand and CAS) of linear functions (in $y = mx+c$ form) and quadratic functions<br>• Transposition (by-hand) of linear functions in A$x$+B$y$=D form to $y=mx+c$ form<br>• Congruent triangles<br>• Translations and dilations on the coordinate plane |
| CAS – enabled techniques to be learned | • Transposition, dilation<br>• Cut-and-paste<br>• Zeroes, $y$-intercepts and points of intersection<br>• General solutions of equations<br>• Area under a curve<br>• Regression |
| VELS dimensions covered | • Structure<br>• Measurement, Chance and Data<br>• Space<br>• Number<br>• Working Mathematically |
| Technology used | Texas Instruments TI-Nspire CAS (v1.6.4295) |
| Lesson Summary | 1. Investigating ◢: Triangle formed by one line and two axes<br>2. Translating the line<br>3. Dilating the line<br>4. A competition: Finding the line given ◢<br>5. Investigating ▲: Triangle formed by two lines and the $x$-axis<br>6. The architect's problem (Assessment Task) |

# Lesson 1
## Investigating ◢: Triangle formed by one line and two axes

<table>
<tr><th>Time (minutes)</th><th>Teacher activity</th><th>Student activity</th><th>Materials</th><th>Student Learning Outcomes</th></tr>
<tr><td>15</td><td>• Introduce unit<br>• Revise by-hand and CAS-enabled sketch graphs of linear and quadratic functions<br>(simple examples only)</td><td>Completion of introductory graphing exercises (see Worksheet, Lesson 1)</td><td rowspan="4">Worksheet<br><br>TI_Nspire CAS calculator<br><br>Graph paper<br><br>PC/laptop with TI-Nspire CAS computer software (Teacher edition) and data projector<br><br>OR<br><br>Overhead projector and viewscreen adaptor</td><td>Previously learned techniques, terminology and knowledge are familiar</td></tr>
<tr><td>10</td><td>• Teach CAS transposition, using solve command; linear only<br>• Teach CAS cut-and-paste techniques for entry of results into Graphs & Geometry screen<br>• Teach CAS graph analysis commands for zero and y-intercept; exact and approximate values</td><td>Completion of Section 1 (see Worksheet, Lesson 1)</td><td>Will be able to sketch and analyse the graph of any linear function using CAS, finding exact and approximate values for gradient and intercepts</td></tr>
<tr><td>20</td><td>• Define ◢ as the area enclosed by the graph of a non-horizontal linear function and the two coordinate axes; simple examples with positive and negative gradients, above and below x-axis.<br>• Teach CAS area measurement technique</td><td>Completion of Section 2 (see Worksheet, Lesson 1)</td><td>Will be able to find ◢ for any linear function, $m \neq 0$<br>Will be able to describe the effects on ◢ of intercepts for specific linear functions</td></tr>
<tr><td>35</td><td>• Introduce first investigation task</td><td>Work on Section 3 investigation task (see Worksheet, Lesson 1)</td><td>Apply knowledge to a problem situation</td></tr>
</table>

Mathematics                                                                                   Name ______________________________

**Linking Linear Functions and Measurement: An investigation using CAS**

## Worksheet 1

### Investigating ◢: Triangle formed by one line and two axes

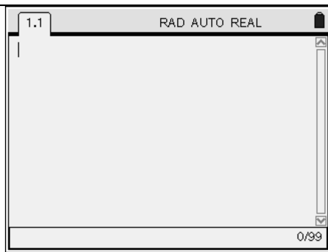
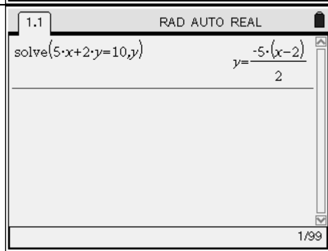
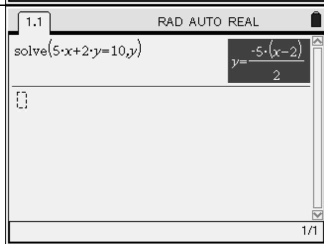
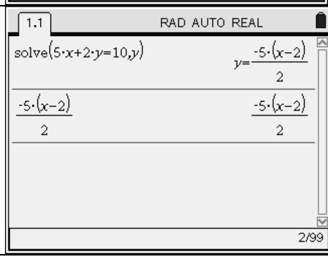
<u>Introductory exercises</u>

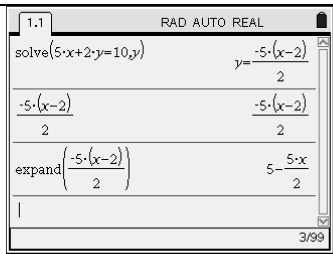
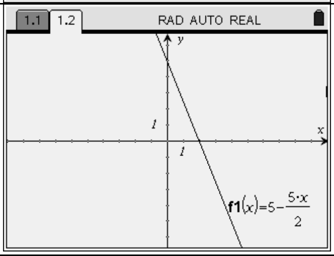
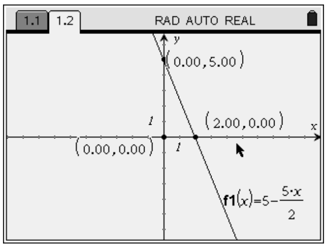
1.Sketch the following linear functions on separate sets of axes using graph paper and your TI-Nspire CAS calculator ( transpose into *y=mx+c* form if necessary):

(a) $y = -2x + 5$          (b) $3x + 5y = 15$          (c) $2y = 3(x - 4)$

<u>Section 1</u>

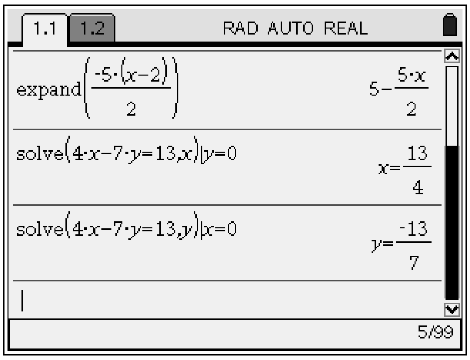
2.Consider the function given by $5x + 2y = 10$.

<table>
<tr><td>Set your calculator to be in automatic mode ([ctrl] 1:File 6:Document Settings), and giving decimal answers to 2 places(Display Digits Fix2) and select the Calculator application.</td><td>
</td></tr>
<tr><td>Access the algebra commands ([menu] 3:Algebra) and choose 1:Solve; type in the function, then a comma, then the pronumeral y and close the brackets, then [enter]; this will enable CAS to transpose your equation into y=mx+c form; or is it?</td><td>
</td></tr>
<tr><td>Scroll up to highlight the new equation.</td><td>
</td></tr>
<tr><td>Press [enter]; scroll left and clear [clear] the y=, making this an expression, and press [enter].</td><td>
</td></tr>
</table>

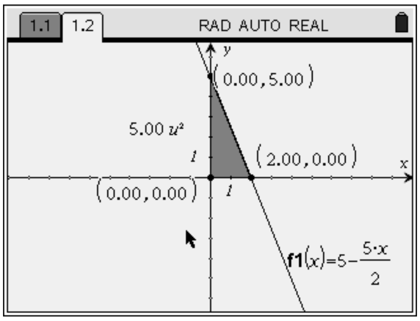
| | |
|---|---|
| Access the algebra commands again and choose 3:expand; scroll up to highlight the expression; press enter and then close the bracket; press enter again.<br><br>You now know its gradient is _______ and its *y*-intercept is______. |  |
| Now scroll up to highlight the expanded expression and press ctrl C to copy the expression; then press home 2 to insert a Graphs & Geometry page;with the cursor across from f1($x$)= press ctrl V and then enter to paste the expression and draw its graph. To hide the entry line press ctrl G. Note the triangle formed by the axes and the line graph. |  |
| Axes intercepts and the origin may be located by finding three pairs of intersections. Press menu 6: Points & Lines then 3: Intersection Point(s). Use the arrow to select (by pressing enter) any pair of lines and the point will appear; repeat for 2 other pairs. Press esc. The decimal (here, exact but can be approximate) coordinates of these points may be found by accessing menu 1: Actions then 7: Coordinates and Equations. Select a point and double-click enter to view the coordinates. Repeat for other 2 points. Press esc. Re-locate these labels if necessary for easier viewing. |  |

3. Now do the same for the functions given by $-3x + 2y = 8$ and $4x - 7y = 13$. As 3 of the 4 axis-intercepts of these lines are not integers, you will need to access the solve function in the Calculator application to view exact fractional coordinates. For an exact $x$-intercept, replace the $y$ of the particular rule with 0, and solve that equation for $x$. Use a similar process for an exact $y$-intercept.

It looks like this for $4x - 7y = 13$:



Section 2

| | |
|---|---|
| We now define ◢ as the area of the triangle enclosed by the axes and the line; using previous knowledge of area and the intercept values just found, calculate the exact value of ◢ for the line given by $5x + 2y = 10$ (use the space to the right for your working out). Also, we will say regardless of the triangle's position above or below the *x*-axis, that ◢ is always positive. | |
| The TI-Nspire CAS calculator will calculate the value of ◢ but only after the triangle has been identified. To do this for $5x + 2y = 10$, return to the previous graph page and press menu then 8: Shapes and 2: Triangle. Select the three vertices you have previously constructed (wait until the "pencil and point" becomes a 'pointed finger" icon before you press enter each time) and the triangle will be recognised.. Now to find ◢ press menu then 7: Measurement and 2: Area. Move the arrow until the triangle is blinking; double-click enter to fix that measurement, and press esc. Re-locate this text (note the *units*² label) if required. Shading is achieved through menu then 1: Actions and 4: Attributes; select the triangle, press enter and the 7 gradations of shading are available through left and right scrolling; press enter and esc to complete the process. |  |
| Now try this for the other 2 lines, $-3x + 2y = 8$ and $4x - 7y = 13$; recall that the Graphs & Geometry application will yield only decimal values (and here, these are *approximate*) while *exact* areas are ensured in the Calculator application by substituting the fractional (but positive) values for base and height. | Area enclosed by the axes and $-3x + 2y = 8$ is _________<br>and the area enclosed by the axes and $4x - 7y = 13$ is _________. |

Section 3

Now conduct an investigation to find the rule/equation for all lines (there are more than one) which have the same value of ◢ as the line $3x - 2y = 6$. The triangles must also be congruent to the original triangle formed by the axes and the line $3x - 2y = 6$. Present all your findings with well-labelled graphs. These may be hand-drawn or sketched using dynamic geometry software (such as Geogebra or Geometer's Sketchpad).

## Lesson 2
### Translating the line

| Time (minutes) | Teacher activity | Student activity | Materials | Student Learning Outcomes |
|---|---|---|---|---|
| 10 | • Recap Lesson 1 findings: definition and calculation of ◢ for any specific linear function, $m \neq 0$<br>• Revision: translations of linear function graphs.<br>(simple examples only) | Completion of translation exercises (see Worksheet, Lesson 2) | Worksheet<br><br>TI_Nspire CAS calculator<br><br>Graph paper<br><br>PC/laptop with TI-Nspire CAS computer software (Teacher edition) and data projector<br><br>OR<br><br>Overhead projector and viewscreen adaptor | Previously learned techniques, terminology and knowledge are familiar |
| 15 | • Teach effect of successive translations (to the right) on ◢ for a specific linear graph.<br>• Teach CAS formulation of quadratic pattern of change in ◢ for horizontal translation of a specific linear graph (use regression to establish function rule). | Completion of Section 1 (see Worksheet, Lesson 2) | | Given a linear graph, will be able to calculate varying values of ◢ (for successive horizontal translations) identifying pattern as quadratic |
| 20 | • Teach CAS generalisation of effect on ◢ for any horizontal translation of a linear graph. | Completion of Section 2 (see Worksheet, Lesson 2) | | Will be able to use CAS and by-hand methods to generalise this pattern for any linear graph |
| 35 | • Introduce second investigation task | Work on Section 3 investigation task (see Worksheet, Lesson 2) | | Apply knowledge to a problem situation |

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**Linking Linear Functions and Measurement: An investigation using CAS**

# Worksheet 2
## Translating the line

Translation exercises

1. By replacing $x$ with $(x - 2)$ in each of the following, write the rule for the image under a translation of → 2 units (2 units to the right) [transpose into $y=mx+c$ form in all cases]:
(*Note: Use CAS for the substitution and transposition of* one *of these, by-hand algebra for the others)*

(a) $y = -2x + 5$ (b) $3x + 5y = 15$ (c) $2y = 3(x - 4)$

Verify your answer for one of the above by sketching the original and the image on the same set of axes, showing horizontal vector arrows of length 2 units between the graphs.

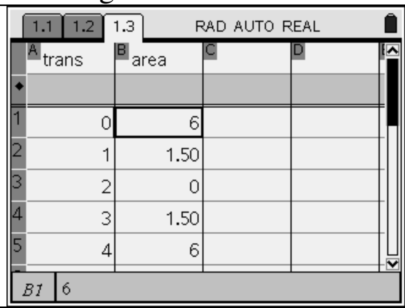
Section 1

2. Consider the function given by $y = 3x + 6$. Complete the table below using CAS and/or by-hand methods:

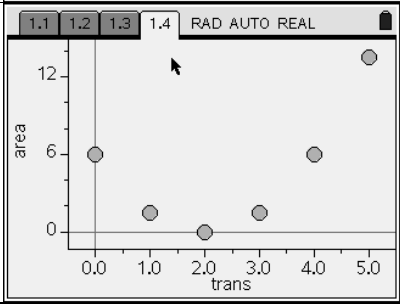
| **Translation** | **Equation ($y = mx + c$ form)** | **Sketch graph** | **Value of ◢** |
|---|---|---|---|
| None | $y = 3x + 6$ | | 6 |
| → 1 unit | | | |
| → 2 units | | | |
| → 3 units | | | |
| → 4 units | | | |
| → 5 units | | | |

3. Now let’s see what happens when CAS is used to analyse these changes:

Insert a Lists & Spreadsheets page by pressing ⌂ 3. Type the headings “trans” and “area” in the top cells in columns A and B. and enter the data (from Section 1 step 2 above) in rows 1 to 6 as shown. (Row 6, not seen here, has entries 5 and 13.50)

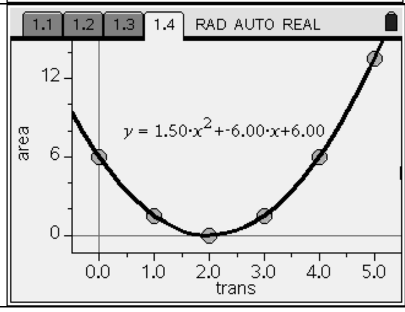



View the scatterplot set up by these 6 data points by pressing ⌂ 5. The points initially will be randomly scattered until you move the arrow to each axis region where “Click to add variable” is located. After clicking, select axis labels as shown. The pattern appears to be quadratic! As column B contained area values, what aspect of area measurement has caused this to happen?




Confirm this pattern by pressing menu then 4: Analyze then 6: Regression and 4: Show Quadratic. The screen shows us the points fit a quadratic equation: $y = \_\_\_ x^2 - \_\_\_ x + \_\_\_$

Which variables do the pronumerals $x$ and $y$ represent here?

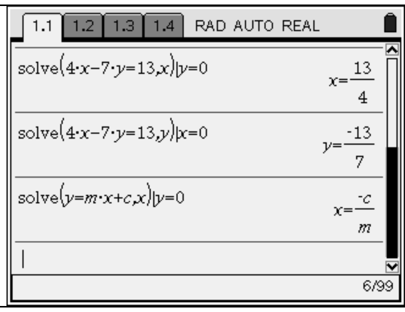



4. This quadratic pattern of area change also works for linear graphs translated *vertically*. Try it for $y = .5x + c$ where $c$ takes on values of −2, −1, 0, 1, and 2 if you have time.

Section 2

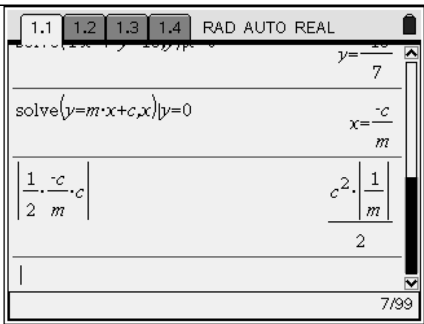
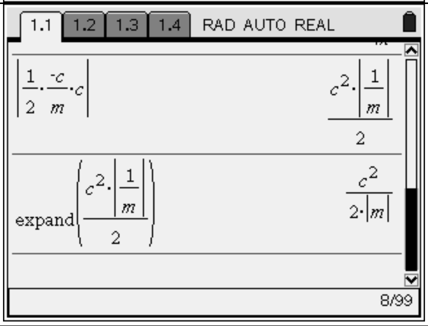
5. But does this work for *all* linear graphs? The algebraic capabilities of the Nspire Calculator application can be used alongside our by-hand methods to investigate this.

Return to the Calculator page. The general linear function $y = mx + c$ will form a triangle whose *height* is $c$ (the $y$-intercept) and whose *base length* (or $x$-intercept) is found by letting $y$=0 and solving for $x$ as shown: (note how the multiplication of $m$ and $x$ must be explicit).




Does CAS make this more or less complicated? ______Will the $x$ intercept $\frac{-c}{m}$ always denote a negative number? ________ Check graphs of lines $y = -3x + 6$, $y = .5x$ and $y = 2x - 8$ to confirm.

As ◢ was defined in Lesson 1 to be non-negative, sign problems are alleviated using the function "absolute value", which eliminates negative signs in simplified answers/expressions. The symbols used for absolute value in by-hand algebra are two vertical lines, and by abs( on the CAS:

| | |
|---|---|
| Using the area formula for a triangle, the value of <u>any</u> linear graph's ◢ is produced originally in a complicated format: |   |
| But expanding this makes it easier to work with:<br><br>Why are there no absolute value signs around the numerator?<br><br>What aspect of this generalised value of ◢ confirms the pattern seen on the graph in Question 3? |   |

6. Using $y = 3x + 6$ and the various new $y$-intercepts from the table you completed in Question 2, verify that this formula generates the values of ◢ that you found previously from the graph.

<u>Section 3</u>

Now conduct an investigation to find the values of ◢ for a set of <u>four</u> parallel lines where the triangle formed is not always above the $x$-axis. The lines should not all be equally spaced. Using quadratic regression, find the rule for the quadratic function which links the $y$-intercept (Column A) with the value of ◢ (Column B). Present all your findings with well-labelled graphs. These may be hand-drawn, sketched on Geogebra, Geometer's Sketchpad or Excel, or presented on the overhead projector adaptor in the next class.

## Lesson 3
### Dilating the line

| Time (minutes) | Teacher activity | Student activity | Materials | Student Learning Outcomes |
|---|---|---|---|---|
| 10 | • Recap Lesson 2 findings: Calculation of ◢ for any general linear function, $m \neq 0$<br>• Revision: dilations of linear function graphs.<br>(simple examples only) | Completion of dilation exercises (see Worksheet, Lesson 3) | Worksheet<br><br>TI_Nspire CAS calculator<br><br>Graph paper<br><br>PC/laptop with TI-Nspire CAS computer software (Teacher edition) and data projector<br><br>OR<br><br>Overhead projector and viewscreen adaptor | Previously learned techniques, terminology and knowledge are familiar |
| 25 | • Teach effect of successive dilations (from $x$-axis) on ◢ for a specific linear graph.<br>• Teach CAS formulation of linear pattern of change in ◢ for vertical dilation of a specific linear graph (use regression to establish function rule). | Completion of Section 1 (see Worksheet, Lesson 3) | | Given a linear graph, will be able to calculate varying values of ◢ (for successive vertical dilations) identifying pattern as linear. |
| 45 | • Guide CAS generalisation of effect on ◢ for any linear graph dilated vertically. | Completion of Section 2 (see Worksheet, Lesson 3) | | Will be able to use CAS and by-hand methods to generalise this pattern for any linear graph |

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**Linking Linear Functions and Measurement: An investigation using CAS**

## Worksheet 3
### Dilating the line

Dilation exercises

1. With the given rule in $y=mx+c$ form (transpose in cases where it isn't), multiply the right side of each of these by 3. Simplify again to $y=mx+c$ form, and write the rule for the image under a vertical dilation by a factor of 3: (*Note: Use CAS for the transposition and expanding of one of these, by-hand algebra for the other two)*
(a) $y = -2x + 5$ (b) $3x + 5y = 15$ (c) $2y = 3(x - 4)$

Verify your answer for one of the above by sketching the original and the image on the same set of axes, showing vertical vector arrows of varying lengths between the graphs. Any point unchanged?

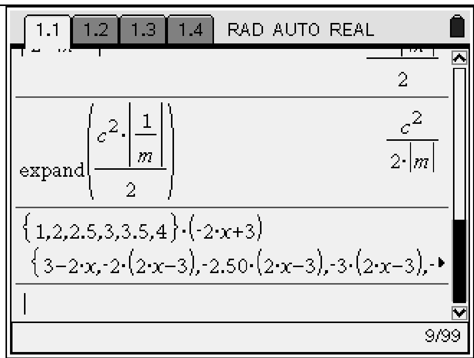
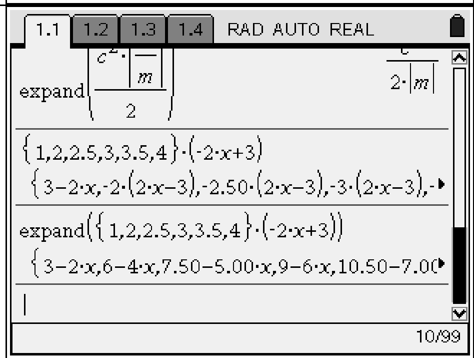
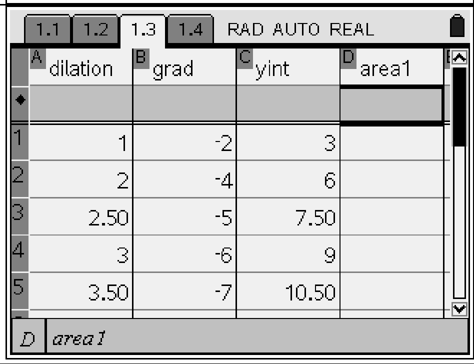
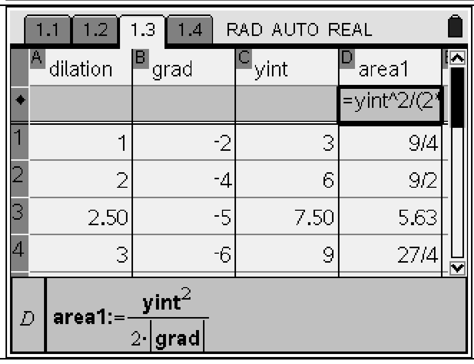
Section 1

2. Consider the function given by $y = ½x + 1$. Complete the table below using CAS and/or by-hand methods:

| **Dilation factor from $x$-axis** | **Equation ($y = mx + c$ form)** | **Sketch graph** | **Value of ◢** |
|---|---|---|---|
| None | $y = ½x + 1$ | | 1 |
| 2 | | | |
| 2.5 | | | |
| 3 | | | |
| 3.5 | | | |
| 4 | | | |

What pattern do you notice? Do you think it holds for all dilated linear graphs?

3. Now look at the effect of dilating $y=-2x+3$ by the same factors; we use CAS to do all the multiplying, expanding, and analysis in these steps:

<table>
<tr>
<td>In the Calculator application, enclose the dilation factors in set (“curly”) brackets, separated by commas, multiplying this group of numbers by the right side of the equation: (some of this is off the screen…)</td>
<td>
</td>
</tr>
<tr>
<td>When you scroll up to the answer, you scroll to the right to read off the 6 expressions, and note they aren’t simplified; so access the 3: Expand command from the Algebra menu; scroll up to the previous input and copy it; don’t forget to close the curved bracket at the end, and press [enter].</td>
<td>
</td>
</tr>
<tr>
<td>Access the Lists & Spreadsheets application. Enter the 6 dilation factors into Column A labelled as “dilation”, the 6 gradients (−2, −4, −5, etc) into Column B labelled as “grad” and the 6 y-intercepts (3, 6, 15/2, etc) into Column C labelled as “yint”; Label the top of Column D as “area1” and position the cursor in the grey region beneath it, as shown.</td>
<td>
</td>
</tr>
<tr>
<td>Get CAS to calculate the value of ◢ for these graphs by pressing [=]yint[x²][÷] (2[×]abs(grad)). This is the formula found in lesson 2, section 2, but with c replaced with “yint” and m replaced with “grad”. Press [enter] and Column D should fill with ◢ values.</td>
<td>
</td>
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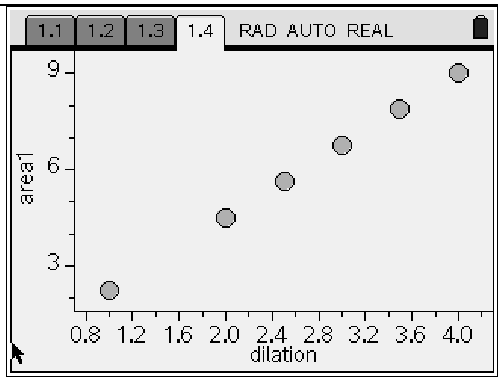
</table>

| | |
|---|---|
| Now by accessing the Data & Statistics application a scattergram comparing ◢ values (area1 on vertical axis) with dilation factors (dilation on horizontal axis) reveals a linear pattern as shown.<br><br>Write the equation of this regression equation as found by CAS (you'll need to follow earlier procedures): |  |

4. If you have time, see if there are similar patterns when you compare gradients (grad) with dilation factors (dilation); also, $y$-intercepts with dilation factors.

<u>Section 2</u>

But does this work for *all* linear graphs? The algebraic capabilities of the CAS home screen can once again be used alongside our by-hand methods to investigate this. Working in teams of 4 (2 using CAS, 2 doing by-hand algebra), and using the letter $k$ to denote the dilation factor from the $x$-axis, investigate the effect of $k$ on the value of ◢ for any line $y = m\text{x} + c$.

# Lesson 4

## A competition: Finding the line given ◢

| Time (minutes) | Teacher activity | Student activity | Materials | Student Learning Outcomes |
|---|---|---|---|---|
| 5 | • Recap Lessons 2 and 3 findings: Influences on ◢ by translations and dilations for any general linear function, $m \neq 0$<br>• Pose questions for discussion (see Worksheet 4) | Discussion within teams | PCs or laptops for student use<br><br>Poster paper<br><br>Worksheet<br><br>Graph paper<br><br>TI_Nspire CAS calculator<br><br>PC/laptop with TI-Nspire CAS computer software (Teacher edition) and data projector<br><br>OR<br><br>Overhead projector and viewscreen adaptor | Leadership and cooperation amongst team members evident as biases for CAS-enabled solutions vs by-hand algebraic solutions are discussed. |
| 60 | • Introduce competition questions, monitoring team dynamics and ensuring engagement of all in the process. (See worksheet 4) | Completion of competitive task within team (see Worksheet, Lesson 3) | | Will be able to monitor own and team's progress towards a solution to the problem.<br><br>Will observe need for flexibility and precision within the team. |
| 15 | • Monitor team presentations.<br>• Respond to presentations | Present results in teams | | Will observe similarities and differences between approaches taken by various teams to solution process. |

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## Worksheet 4
### A competition: Finding the line given ◢

<u>Discussion Questions</u>

1. In your team of four students, discuss the variety of ways the value of ◢ can be altered for any given line, and the effect of that alteration.

2. If you knew in advance what the value of ◢ was, how might you as a team find the equation of the line that produces that area? Will there be only one solution? What methods (including different technologies) might be best used within the team? Should all work together, or should individuals choose their own procedure? How might the results be presented effectively?

<u>Competition</u>

3. Out of the following numbers choose one for your team:

   534 620 56.5 146 22.7 0.4 134

   This is your team's value of ◢.

   In as many ways as possible, find a line or a group of lines which have this particular value of ◢.

   You have one hour to work collaboratively to solve the problem and prepare a presentation to your classmates. The presentation will be judged on the following criteria:

   - Observed cooperation between team members
   - Accuracy of solution
   - Variety of representations of problem and solution (*ie* graphs, technology, algebra, etc)
   - Use of correct terminology and notation in presentation.

# Lesson 5

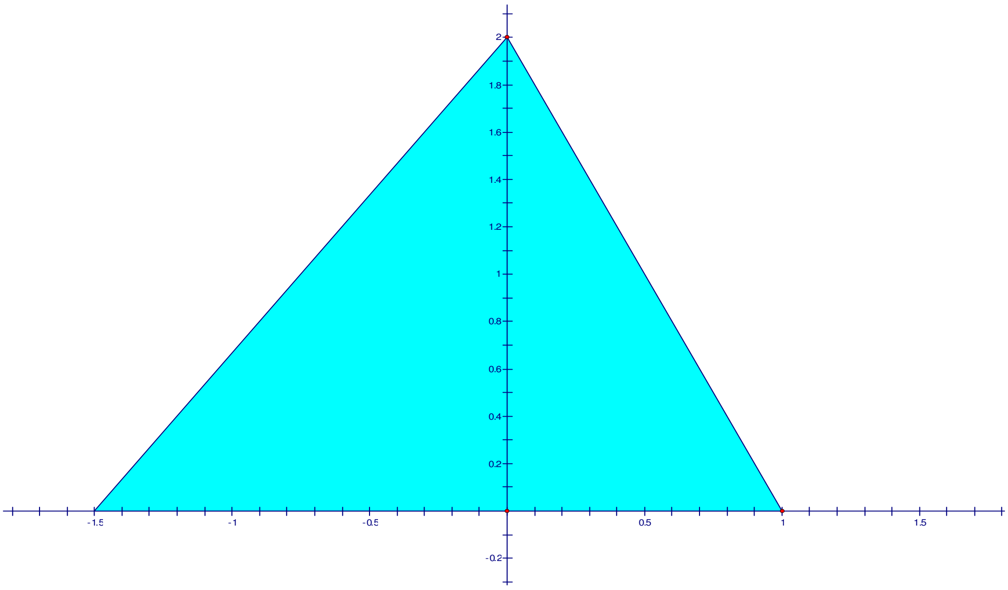
## Investigating ▲: Triangle formed by two lines and the *x*-axis

| Time (minutes) | Teacher activity | Student activity | Materials | Student Learning Outcomes |
|---|---|---|---|---|
| 10 | • Define ▲ as the area enclosed between the *x*-axis and two non-horizontal lines. One of these two lines must have a positive gradient, and the other a negative gradient. (Simple example to illustrate) | Completion of Section 1 (see Worksheet, Lesson 5) | Worksheet<br><br>TI_Nspire CAS calculator<br><br>Graph paper<br><br>PC/laptop with TI-Nspire CAS computer software (Teacher edition) and data projector<br><br>OR<br><br>Overhead projector and viewscreen adaptor | Calculation of ▲ for two lines whose *y*-intercepts are equal. |
| 10 | • What happens if the *y*-intercepts are not equal? Locating the common height. (Simple example to illustrate) | Finish Section 2 with teacher guidance (see Worksheet, Lesson 5) | | Calculation of ▲ for two lines whose *y*-intercepts are not equal. |
| 30 | • Introduce problem from Section 3 of worksheet; show use of CAS to formulate the value of ▲ in terms of *m*, the unknown gradient. | Completion of Section 3 with teacher guidance (see Worksheet, Lesson 5) | | Will be able to find *m* for any linear function whose *y*-intercept is known, for any other given linear function and a given value of ▲. CAS formulation and solving of equations involving *m* will be privileged. |
| 30 | • Introduce fifth investigation task | Work on Section 4 investigation task (see Worksheet, Lesson 5) | | Apply knowledge to create a problem situation, similar to that seen in Section 3, where the value of ▲ is known, one line is completely defined, and the other line has a given gradient but an unknown *y*-intercept. |

Mathematics                                        Name ______________________________

**Linking Linear Functions and Measurement: An investigation using CAS**

## Worksheet 5
### Investigating ▲: Triangle formed by two lines and the *x*-axis



<u>Section 1</u>

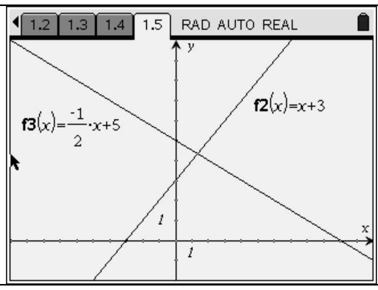
Consider the triangle formed by two lines and the $x$-axis as shown above. Find the equations of each of the lines in $y=mx+c$ form; also, find the value of ▲, the area enclosed between the $x$-axis and two non-horizontal lines. Use

(a) the area formula $A=\dfrac{bh}{2}$;
(b) the individual values of ◢ for each line, using their $m$ and $c$ values;
(c) the overall value of ▲ for the lines, using the measurement command in the Graphs & Geometry application on the CAS calculator.

<u>Section 2</u>

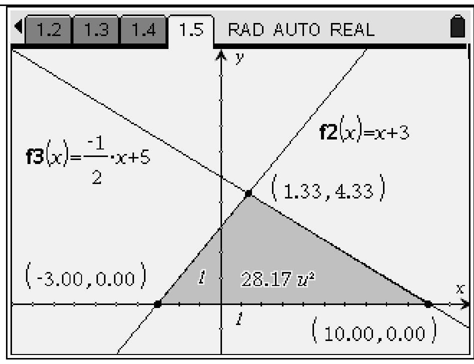
Now let's see what happens when the $y$-intercepts of the given lines are not equal. Consider the problem of finding ▲ for $y = x + 3$ and $y = -½x + 5$.

On a new Graphs & Geometry page, enter the two functions. The graph to the right is formed by a window where the $x$-values range from $-10$ to 12, and the $y$-values range from $-2$ to 10. The scales on both axes are 1 unit each.



The most obvious method of finding an *approximate* value of ▲ is to use the measurement command, introduced in Lesson 1 Section 2. The screen shown here will remind you of the setup required, involving intersection points of the 2 lines and the *x*-axis.

Correct to 2 decimal places the value of ▲ is _____________.



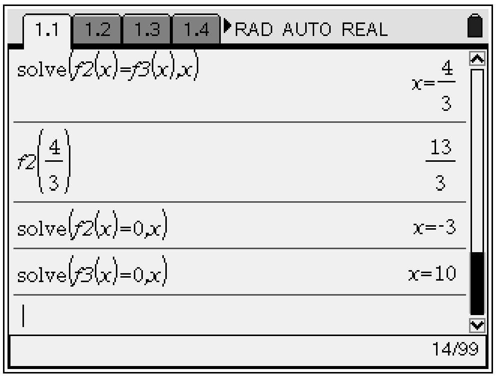

We can calculate this value in a more traditional way by examining the coordinates of the 3 vertices. Which of these give information about the height of the triangle? __________ Which of them tell us about the base?

______________________________

In the space at the right, calculate ▲ using the formula for area of a triangle.

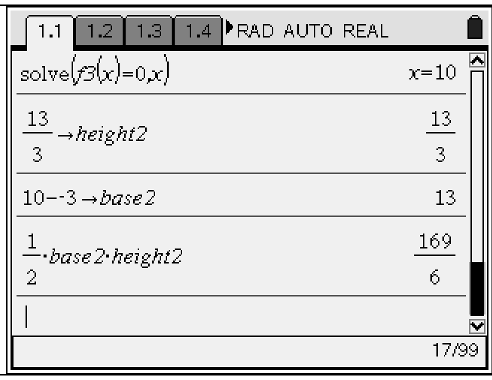
You may have found a slight difference in the answers obtained using CAS measurement as compared with using coordinate values in the formula.

Exact values of the key coordinates required for the height and base can be found in the Calculator application as shown on the right. Note the use of the symbols f2($x$) and f3($x$) as being a more efficient way of naming the linear functions.



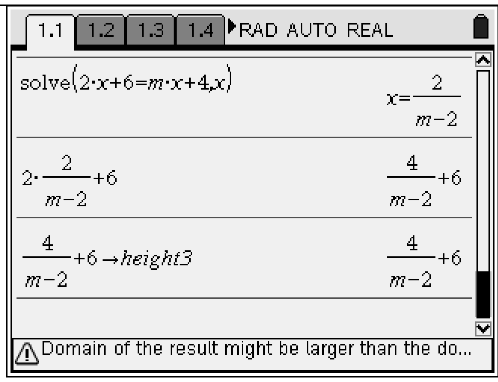

Storing new variables (using ctrl sto→) "base2" and "height2" enables you to complete the calculation of ▲ as an exact value. See screen at right.

Is this consistent with the value 28.17 found earlier?



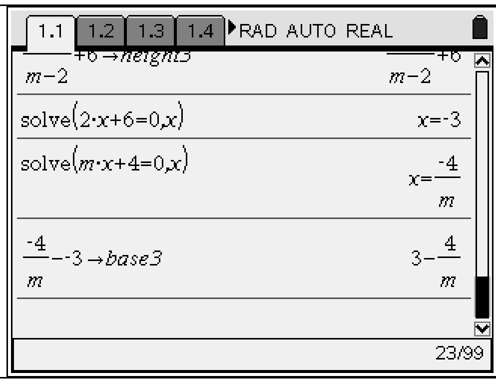
Section 3

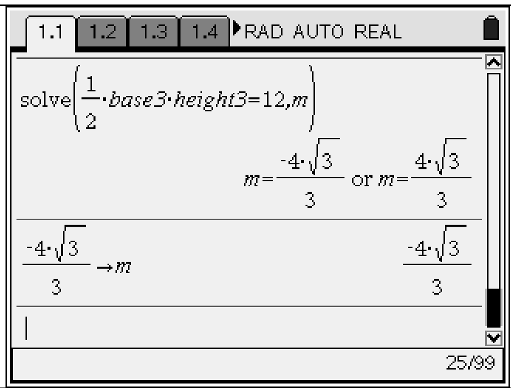
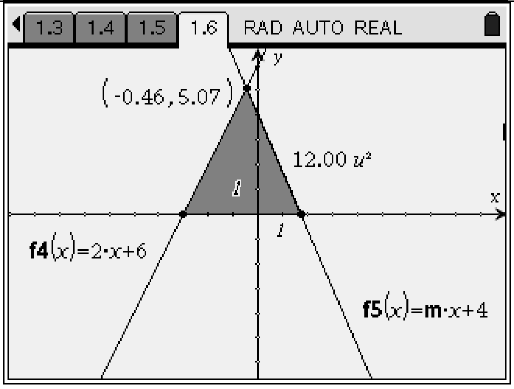
We now consider the task of having a given ▲ value, and trying to find the gradient of one of the two lines.
Suppose we wish to make ▲ = 12 when one of the lines is $y = 2x + 6$ and the other line is $y = mx + 4$.

<table>
<tr>
<td>From the work in Section 2, it can be seen that the task requires us to find the point of intersection of the 2 lines. Use the solve function to set the two "right-hand sides" equal to each other (as they both equal y) and solve for x, in terms of m. [Note again the need to make the multiplication of m and x explicit]. The y-coordinate of this point is then found by substitution into $y = 2x + 6$, and stored as "height3" as shown.</td>
<td>
</td>
</tr>
</table>

The warning at the bottom of the screen alerts us to the fact that not all values of m are permissible here. What would happen if m=2? How would this impact on the task of creating a triangle?

The base of the overall triangle is found by subtraction of the $x$ intercepts of the line $y = 2x + 6$ and $y = mx + 4$ ; these are $-3$ and $\frac{-4}{m}$, respectively. Since the latter is actually positive (why?) the order of subtraction should be $\frac{-4}{m} - (-3)$ , or $\frac{3m-4}{m}$; this too is a positive number. The height of the triangle is the previously found $y$-coordinate of the point of intersection, $\frac{4}{m-2} + 6$.

<table>
<tr>
<td>Of course, this can all be done on CAS. Note the inputs required for the x intercepts of the two lines to be found, subtracted, and the result stored as "base3"…</td>
<td>
</td>
</tr>
</table>

| | |
|---|---|
| We can complete the procedure by setting the total area equal to 12 and then solving for *m*, as required.<br><br>Note the two values for *m*. Why was the negative one chosen for storage as the true value of *m*? |  |
| As a check, a new Graphs & Geometry page can be utilised to confirm the two lines have a ▲ value of 12. Note how in the example shown, the stored numerical value of *m* will be used in function f5($x$).<br><br>Explain how you know whether the triangle is isosceles or not: |  |

Section 4

This investigation will involve using the CAS to create two lines (one fully defined, and one whose gradient is known to be negative but whose *y*-intercept is unknown), where ▲ is known. The only restriction is that you cannot use the lines used in Section 3, and the value of ▲ must also be a number between 20 and 40.

To help you get started, make these decisions:

My value of ▲ is ______________. My completely defined line equation is ________________________ (remember, its gradient is negative).
My other line has a gradient of ________ so its equation is $y$ = ___$x$ + c. (This gradient must be positive)

A rough sketch of the situation on graph paper, knowing the triangle has to have a specific area, will help you to adjust the window settings of your CAS.

Results:

## Lesson 6

### The Architect’s Problem (Assessment Task)

| Time (minutes) | Teacher activity | Student activity | Materials | Student Learning Outcomes |
|---|---|---|---|---|
| 80 | Introduce The Architect’s Problem<br><br>Answer questions as required | Begin work on Architect’s Problem | Worksheet<br><br>TI_Nspire CAS calculator<br><br>Graph paper<br><br>PC/laptop with TI-Nspire CAS computer software (Teacher edition) and data projector<br><br>OR<br><br>Overhead projector and viewscreen adaptor | Students will apply knowledge gained in previous sessions to solve a practical problem. |

Mathematics Name ______________________________

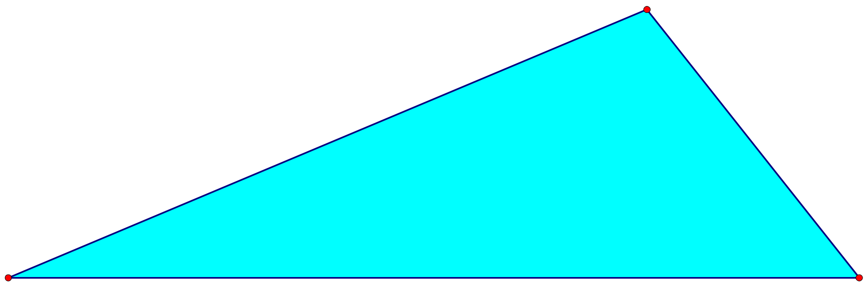
**Linking Linear Functions and Measurement: An investigation using CAS**

# Worksheet 6

## The Architect's Problem

A major international corporation is building a new headquarters in Melbourne. They wish to engage the services of a leading local architect to design a large structure that will sit in a prominent place in front of the building.

Required to be shaped as a scalene triangle, with its longest side horizontal, the area of the front face of the structure must be 50m². The angles may not necessarily be identical to the diagram below, but the right side must be the shortest of the 3 sides.



The architect needs to plan her work with care. She is using a mathematical software program to design the sign. Linear graphs represent the sloping sides; while the base is represented by the *x*-axis. Each unit used on the coordinate grid will represent 1 metre.

Your task, as the architect's assistant designer, is to find the equations she will need to enclose an area of 50m². You must be able to demonstrate, with calculations and diagrams, that you have completed the task successfully. This will be done at a board meeting next week with the architect and her client in attendance.

* * * * * * * * * * * * * * * * * * * * * * * * *

The work will be assessed using similar criteria to those used in the Lesson 4 investigation:

- Observed application to task within class
- Evidence of further work outside class
- Accuracy of solution
- Variety of representations of problem and solution (*ie* graphs, technology, algebra, etc)
- Use of correct terminology and notation in presentation.

# Commentary on Curriculum Unit

# Linking Linear Functions and Measurement:Investigating using CAS

## Introduction

In writing this unit of work I wanted to create an idea that was interesting to teachers as well as to students. Instead of recycling well-worn textbook ideas, I felt it was important to produce a body of material that utilised the diverse range of representations available on a CAS calculator. I have been interested in algebraic representations of geometric ideas for years, and found that I was discovering answers to my own "what if" questions through experimentation. Thus, students throughout the unit are challenged to create, explore, explain and apply the mathematical concepts contained within.

Part of the creative process included the new concepts of ◢ and ▲, which I hope would be given names by the students and teachers who use them. Calculated as specific values early in the unit, they become complex functions of the lines (and their transformations) that form them. The constant "what if…?" posed in the unit serves to remind students and teachers that there are no limits to mathematical thought, and that creativity is now, more than ever before, in the hands of young people. CAS in its "white box" function is an essential part of this process but by-hand methods are recognised as at times being useful and efficient as well.

Geometric, algebraic, numerical and statistical features of the CAS need to be seen as inherently related to each other. Middle secondary teachers who are not mathematics specialists are often likely to keep each syllabus topic in its own little box instead of being part of a much more grand scheme; this often reflects the way they themselves were taught. In the spirit of the VELS "Working Mathematically" domain, the interrelationships of mathematical ideas are front and centre of this unit of work. It is hoped that the step-by-step nature of some of the CAS procedures in this unit can provide a basis for scaffolding mathematical knowledge for teachers as well as for students.

## General commentary

Teaching methods in the first five lessons of this unit feature presentations of CAS-specific techniques which will be new to students. As the teacher is expected to use an overhead projector adaptor, she/he can either ask students to keep their screens looking like that which is on the wall, or students can be invited to show/explain their answers. The class thus has an opportunity to diagnose any errors. This again reflects the "teacher as mentor" role and the changing effect on classroom didactics. When students begin their independent or group work in the lesson, the teacher can refer to specific CAS-based skills on the appropriate worksheet which might enable confused students to get back on track.

Team work features strongly in Lesson 4 where the emphasis is on utilisation of the combined skill level of the members. Middle secondary students enjoy competition as long as they feel the contest is fair. They are given a choice in what ◢ value they can use and can nominate the degree to which they use technology to assist them. The teacher needs to provide a variety of resources (as listed) so that the various strengths of team members can be utilised.

Individual independent work is required in the assessment task which is to start in Lesson 6, and might well continue in subsequent lessons. It was felt that by this point, having experienced whole-class teaching and peer group problem-solving strategies, the students will have a full complement of possible strategies for solving the problem. As each student's solution will be unique, the teacher can be more confident that any "out of class" discussions will not provide the solution, but rather a general pathway to that solution.

## Lesson-by-lesson commentary

> Note: Each lesson features a worksheet. The teacher will guide the students through the various questions posed and the new CAS techniques described and illustrated with screen dumps. Each ends with an individual or group task which should be completed before the next lesson. Also, it must be stated that given the nature of most schools' timetables, the six lessons allocated to this unit of work might take up to three weeks to present if the class is of mixed abilities. Absences and the usual "I don't have my calculator" mantra make an extended time frame more realistic.

## Lesson 1

CAS-assisted graph analysis is presented by the teacher and ◢ is defined as the area of a triangle bounded by a non-horizontal line and the two coordinate axes. The varying value of ◢ is seen to be dependent on the line. The measurement tool within the Algebra menu is employed as a quick method of calculating ◢ for CAS-produced graphs. The investigation at the end of the lesson is designed to allow students to explore the symmetry of such triangles produced by parallel lines and their reflections in the axes. As with other lessons, the work is rich with terminology and geometric visualisation of algebraic concepts.

## Lesson 2

Revision of the previous lesson's work begins this and all remaining lessons, in an effort to produce familiarity with the techniques as well as the location of the key menu items on this CAS. Horizontal translations are seen to produce a quadratic effect in the value of ◢ which is evident in numeric, symbolic and graphical representations – a critical feature of any CAS. The investigation involves vertical translations without naming them as such, and invites students to use other technologies for their solutions – again, essential in opening up multiple solution pathways.

## Lesson 3

The focus of this lesson is dilations. By-hand and/or CAS methods are encouraged as the teacher emphasises the need for a balanced, multi-faceted approach for solutions. Multiple simplifications on the home screen and linear regression are used here, with further pattern exploration invited for more motivated students. A very open-ended investigation enables students to generalise in a manner similar to that done in Lesson 2.

**Lesson 4**

A competition amongst teams of four students predominates this lesson. Students are challenged to decide roles and approaches which satisfy the goal of producing multiple pathways to a solution. The problem posed is one where ◢ is given but the lines are to be found. This is felt to be a most important activity as reversing the "given/to find" order is an essential skill in higher mathematics. The experience will assist individual students in preparing for the overall assessment task in Lesson 6. The teacher provides a number of different technologies for students use. Assessment criteria as stated give a clear signal to students that they have responsibility to the group. When solutions are given in short presentations at the conclusion of the lesson, the dissemination of various solution strategies will be valuable and creativity will be recognised.

**Lesson 5**

In a leadup to the overall assessment, ▲ is defined to be the area of the triangle formed by two non-parallel lines and the *x*-axis. The limitations of some CAS techniques are pointed out here and in other lessons, and students are challenged to make an informed choice based on all knowledge they possess. Discernment between two distinct CAS-enabled solutions is required, another essential skill for today's students. The investigation features student choice of a ▲ value and a subsequent search for two lines, one of which is completely defined and the other where the gradient is unknown. CAS-enabled solutions of difficult equations are seen throughout the lesson.

**Lesson 6**

The main assessment task dominates this lesson. Students take on a real-world problem involving area and apply the skills and techniques learned previously to arrive at a solution which they must justify in multiple ways. The link between theory and real-world measurement is emphasised so that true meaning can be derived from the work.

## Conclusion

The use of CAS at most secondary schools has filtered down to Years 9 and 10. Golden opportunities exist to make the algebra come to life for and mean something to more students. As senior mathematics examiners are re-evaluating exam questions in light of what CAS can do, middle secondary teachers also need to question the isolated manner in which topics are often taught. Integrating various branches of mathematics into problem-solving can give new and deeper understanding to our students. CAS use enables these younger students to access rich mathematical activities. Hopefully this body of work will make a contribution to that effort.

Roger Wander