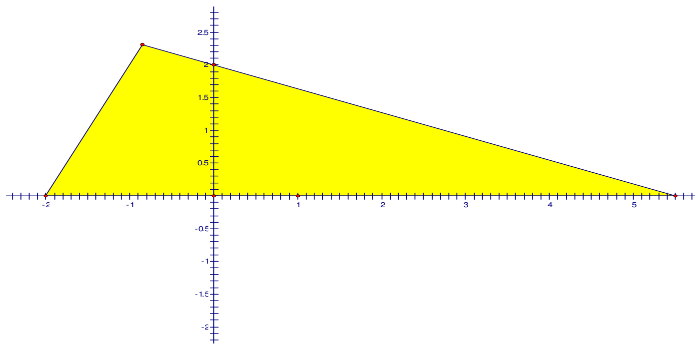
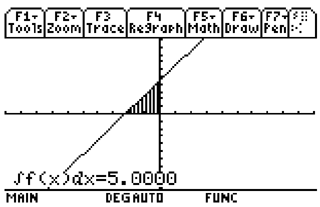
Linking Linear Functions and Measurement: Investigating using CAS

(TI-89 version)





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The University of Melbourne
January 2009

Contents

Background to the Unit of Work

Year Level	10 (with 1+ years' exposure to CAS with varying degrees of use)
Time allocation	6 lessons, comprising normal class meeting times in one 10-day cycle with 4 periods (assume 80 minutes each) per day.
Assumed Knowledge	• Sketch graphs (by-hand and CAS) of linear functions (in $y = mx+c$ form) and quadratic functions • Transposition (by-hand) of linear functions in Ax+By=D form to $y=mx+c$ form • Congruent triangles • Translations and dilations on the coordinate plane
CAS – enabled techniques to be learned	• Transposition, dilation • Cut-and-paste • Zeroes, y-intercepts and points of intersection • General solutions of equations • Area under a curve • Regression
VELS dimensions covered	• Structure • Measurement, Chance and Data • Space • Number • Working Mathematically
Technology used	Texas Instruments TI-89 Titanium (OS v3.10)
Lesson Summary	1. Investigating ◢: Triangle formed by one line and two axes 2. Translating the line 3. Dilating the line 4. A competition: Finding the line given ◢ 5. Investigating ▲: Triangle formed by two lines and the x-axis 6. The architect's problem (Assessment Task)

Lesson 1

Investigating ◢: Triangle formed by one line and two axes

Time (minutes)	Teacher activity	Student activity	Materials	Student Learning Outcomes
15	• Introduce unit • Revise by-hand and CAS-enabled sketch graphs of linear and quadratic functions (simple examples only)	Completion of introductory graphing exercises (see Worksheet, Lesson 1)	Worksheet CAS calculator Graph paper Overhead projector and viewscreen adaptor	Previously learned techniques, terminology and knowledge are familiar
10	• Teach CAS transposition, using solve(command; linear only • Teach CAS cut-and-paste techniques for entry of results into f_1= screen • Teach CAS graph analysis commands for zero and *y*-intercept; exact and approximate values	Completion of Section 1 (see Worksheet, Lesson 1)		Will be able to sketch and analyse the graph of any linear function using CAS, finding exact and approximate values for gradient and intercepts
20	• Define ◢ as the area enclosed by the graph of a non-horizontal linear function and the two coordinate axes; simple examples with positive and negative gradients, above and below *x*-axis. Teach CAS area under a curve technique	Completion of Section 2 (see Worksheet, Lesson 1)		Will be able to find ◢ for any linear function, $m \neq 0$ Will be able to describe the effects on ◢ of intercepts for specific linear functions
35	• Introduce first investigation task	Work on Section 3 investigation task (see Worksheet, Lesson 1)		Apply knowledge to a problem situation

Mathematics Name ______________________________

Linking Linear Functions and Measurement: An investigation using CAS

Worksheet 1
Investigating ◢: Triangle formed by one line and two axes

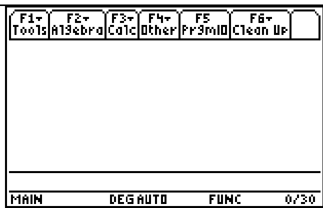
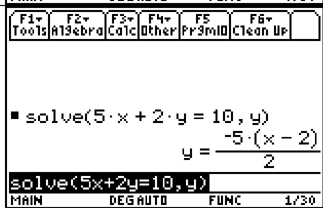
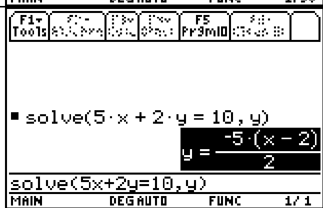
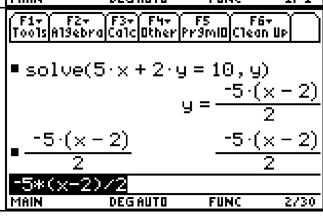
Introductory exercises

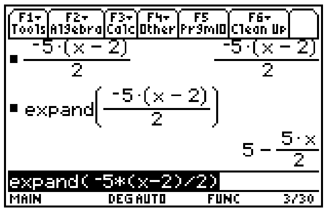
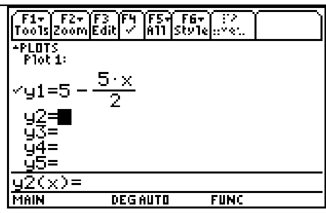
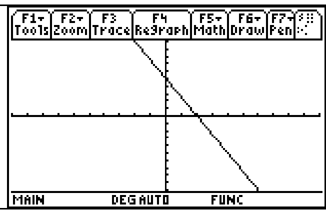
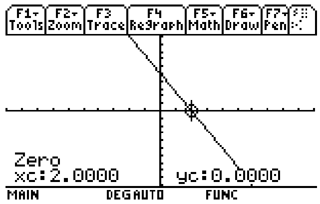
1.Sketch the following linear functions on separate sets of axes using graph paper and your TI-89 CAS calculator (transpose into $y=mx+c$ form if necessary):

(a) $y = -2x + 5$ (b) $3x + 5y = 15$ (c) $2y = 3(x - 4)$

Section 1

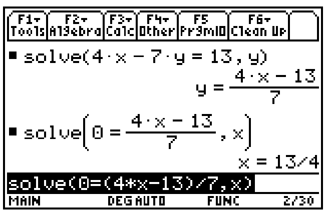
2.Consider the function given by $5x + 2y = 10$.

Set your calculator to be in automatic mode (MODE F2), and giving decimal answers to 4 places(MODE, display digits FIX4) and clear the home screen (HOME F1 8:ClearHome)	
Access the algebra commands by pressing F2; choose 1:Solve(; type in the function, then a comma, then the pronumeral *y* and close the brackets; this will enable CAS to transpose your equation into *y=mx+c* form; or is it?	
Scroll up to highlight the new equation	
Press ENTER; scroll right to de-highlight the equation; scroll left and eliminate the y=, making this an expression, and press ENTER	

Access the algebra commands again (F2) and choose 3:expand(; scroll up to highlight the expression; press ENTER and then close the bracket; press ENTER again. You now know its gradient is _______ and its *y*-intercept is______.	
Now scroll up to highlight the expanded expression and press ↑ to copy the expression; then press F1 to access the function graphing menu; with the cursor across from Y_1= press ESC and then ENTER to paste the expression.	
Access the zooming feature by pressing F2 and then pressing 6:ZoomStd to view the graph with the axes both set in the intervals −10 to +10. Note the triangle formed by the axes and the line graph.	
Axes intercepts may be found by accessing the Math commands at F5 and then 1:Value and typing 0 (the *x*-coordinate of the *y*-intercept), and then F5 and 2:Zero for the *x*-intercept; you will be asked to name, or scroll with the cursor to, a "lower bound" somewhere to the left of the *x*-intercept; after having done so, a similar request for an "upper bound" is given; this just cuts down the time spent in finding the *x*-intercept; pressing ENTER at the end of this process produces the screen as shown.	

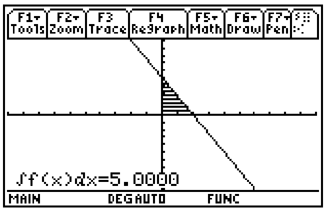
3. Now do the same for the functions given by $-3x + 2y = 8$ and $4x - 7y = 13$. As 3 of the 4 axis-intercepts of these lines are not integers, you will need to access the solve(function – on the home screen! For an exact *x*-intercept, replace the *y* of the particular *y* = form with 0, and solve that equation for *x*.

It looks like this for $4x - 7y = 13$:



Do a similar process for the exact *y*-intercept.

Section 2

We now define ◢ as the area of the triangle enclosed by the axes and the line; using previous knowledge of area and the intercept values just found, calculate the exact value of ◢ (use the space to the right for your working out):	
The TI-89 CAS calculator has a function which calculates the area between a function and the x-axis; access it using the Math commands (F5) and then 7:∫f(x)dx; this is a calculus command but a handy one for you in this investigation; you'll be asked for the lower limit (type in 0, the x-coordinate of the left-most part of the triangle, then ENTER) and then a request for an upper limit (type in the x-intercept, the right-most x-coordinate of the triangle, which is 2) press ENTER and you'll see the shading and the area.	
Now try this for the other 2 lines, $-3x + 2y = 8$ and $4x - 7y = 13$; be sure to enter *exact* values for the lower and upper limits of x. Note that, for $4x - 7y = 13$, the triangle is <u>below</u> the x-axis and the area given by CAS is negative; for our purposes, we'll treat the area as a positive quantity regardless of where it is.	Area enclosed by the x-axis and $-3x + 2y = 8$ is ________ and the area enclosed by the x-axis and $4x - 7y = 13$ is ________.

Section 3

Now conduct an investigation to find the rule/equation for all lines (there are more than one) which have the same value of ◢ as the line $3x - 2y = 6$. The triangles must also be <u>congruent</u> to the original triangle formed by the axes and the line $3x - 2y = 6$. Present all your findings with well-labelled graphs. These may be hand-drawn or sketched using dynamic geometry software (such as Geogebra or Geometer's Sketchpad).

Lesson 2
Translating the line

Time (minutes)	Teacher activity	Student activity	Materials	Student Learning Outcomes
10	• Recap Lesson 1 findings: definition and calculation of ◢ for any specific linear function, $m \neq 0$ • Revision: translations of linear function graphs. (simple examples only)	Completion of translation exercises (see Worksheet, Lesson 2)	Worksheet CAS calculator Graph paper Overhead projector and viewscreen adaptor	Previously learned techniques, terminology and knowledge are familiar
15	• Teach effect of successive translations (to the right) on ◢ for a specific linear graph. • Teach CAS formulation of quadratic pattern of change in ◢ for horizontal translation of a specific linear graph (use regression to establish function rule).	Completion of Section 1 (see Worksheet, Lesson 2)		Given a linear graph, will be able to calculate varying values of ◢ (for successive horizontal translations) identifying pattern as quadratic
20	• Teach CAS generalisation of effect on ◢ for any horizontal translation of a linear graph.	Completion of Section 2 (see Worksheet, Lesson 2)		Will be able to use CAS and by-hand methods to generalise this pattern for any linear graph
35	• Introduce second investigation task	Work on Section 3 investigation task (see Worksheet, Lesson 2)		Apply knowledge to a problem situation

Mathematics Name ______________________________

Linking Linear Functions and Measurement: An investigation using CAS

Worksheet 2
Translating the line

Translation exercises

1. By replacing x with $(x - 2)$ in each of the following, write the rule for the image under a translation of → 2 units (2 units to the right) [transpose into $y=mx+c$ form in all cases]:
(*Note: Use CAS for the substitution and transposition of one of these, by-hand algebra for the others)*

(a) $y = -2x + 5$ (b) $3x + 5y = 15$ (c) $2y = 3(x - 4)$

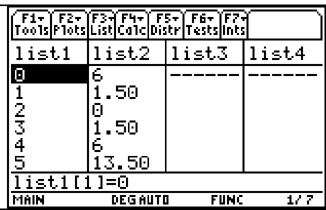
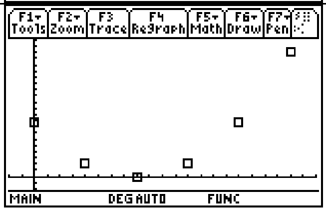
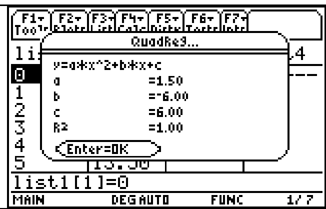
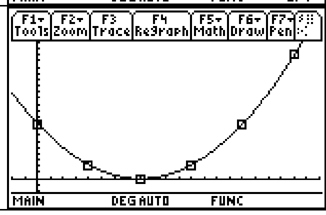
Verify your answer for one of the above by sketching the original and the image on the same set of axes, showing horizontal vector arrows of length 2 units between the graphs.

Section 1

2. Consider the function given by $y = 3x + 6$. Complete the table below using CAS and/or by-hand methods:

Translation	**Equation ($y = m$x $+ c$ form)**	**Sketch graph**	**Value of ◢**
None	$y = 3x + 6$		6
→ 1 unit			
→ 2 units			
→ 3 units			
→ 4 units			
→ 5 units			

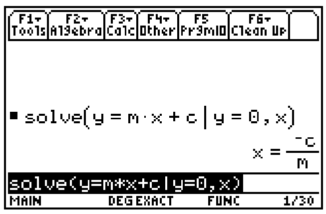
3. Now let's see what happens when CAS is used to analyse these changes:

Set calculator to Auto mode and Fix2 for rounding. Clear any functions out of Y= screen. Using the Stats – List Editor icon found under Apps, enter the table data (see step2, previous page) from the first column into list1 and the data from the last column into list2.	
Vew the scatterplot set up by these 6 data points by pressing F2(Plots), then 1:Plot Setup, then F1(Define). Choose Scatter (Plot Type), Box (Mark), let *x* and *y* be list1 and list2, respectively; press ENTER. The pattern appears to be quadratic! As list2 contained area values, what aspect of area measurement has caused this to happen?	
Navigate back to the lists. Confirm this pattern by pressing the F4 (Calculate) key and pressing 3: Regressions and then 4:QuadReg; the screen shows us the points perfectly (as R²=1) fit a quadratic equation: $y = \_\_\_ x^2 - \_\_\_ x + \_\_\_$	
Navigating back to the lists and pressing F3, the graph confirms our observations	

4. This quadratic pattern of area change also works for linear graphs translated *vertically*. Try it for $y = .5x + c$ where *c* takes on values of −2, −1, 0, 1, and 2 if you have time.

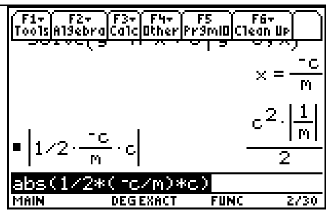
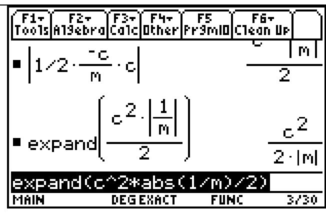
<u>Section 2</u>

5. But does this work for *all* linear graphs? The algebraic capabilities of the CAS home screen can be used alongside our by-hand methods to investigate this.

Press F6 1: to clear any pronumeral values, and choose the exact mode. The general linear function $y = mx + c$ will form a triangle whose height is *c* and whose *x*-intercept (or *base length*) is found by letting *y*=0 and solving for *x* as shown: (note how the multiplication of *m* and *x* must be explicit).	

Does CAS make this more or less complicated? ______Will the *x* intercept $\frac{-c}{m}$ always denote a negative number? ________ Check graphs of lines $y = -3x + 6$, $y = .5x$ and $y = 2x - 8$ to confirm.

As ◢ was defined in Lesson 1 to be non-negative, sign problems are alleviated using the function “absolute value”, which eliminates negative signs in simplified answers/expressions. The symbols used for absolute value in by-hand algebra are two vertical lines, and by abs(on the CAS:

Using the area formula for a triangle, and finding abs(in the catalogue, the value of <u>any</u> linear graph’s ◢ is produced originally in a complicated format:	
But expanding this makes it easier to work with: Why are there no absolute value signs around the numerator? What aspect of this generalised value of ◢ confirms the pattern seen on the graph in Question 3?	

6. Using $y = 3x + 6$ and the various new y-intercepts from the table you completed in Question 2, verify that this formula generates the values of ◢ that you found previously from the graph.

<u>Section 3</u>

Now conduct an investigation to find the values of ◢ for a set of <u>four</u> parallel lines where the triangle formed is not always above the x-axis. The lines should not all be equally spaced. Using quadratic regression, find the rule for the quadratic function which links the y-intercept (list1) with the value of ◢ (list2). Present all your findings with well-labelled graphs. These may be hand-drawn, sketched on Geogebra, Geometer’s Sketchpad or Excel, or presented on the overhead projector adaptor in the next class.

Lesson 3
Dilating the line

Time (minutes)	Teacher activity	Student activity	Materials	Student Learning Outcomes
10	• Recap Lesson 2 findings: Calculation of ◢ for any general linear function, $m \neq 0$ • Revision: dilations of linear function graphs. (simple examples only)	Completion of dilation exercises (see Worksheet, Lesson 3)	Worksheet CAS calculator Graph paper Overhead projector and viewscreen adaptor	Previously learned techniques, terminology and knowledge are familiar
25	• Teach effect of successive dilations (from x-axis) on ◢ for a specific linear graph. • Teach CAS formulation of linear pattern of change in ◢ for vertical dilation of a specific linear graph (use regression to establish function rule).	Completion of Section 1 (see Worksheet, Lesson 3)		Given a linear graph, will be able to calculate varying values of ◢ (for successive vertical dilations) identifying pattern as linear.
45	• Guide CAS generalisation of effect on ◢ for any linear graph dilated vertically.	Completion of Section 2 (see Worksheet, Lesson 3)		Will be able to use CAS and by-hand methods to generalise this pattern for any linear graph

Mathematics   Name ______________________________

Linking Linear Functions and Measurement: An investigation using CAS

Worksheet 3
Dilating the line

Dilation exercises

1. With the given rule in $y=mx+c$ form (transpose in cases where it isn't), multiply the right side of each of these by 3. Simplify again to $y=mx+c$ form, and write the rule for the image under a vertical dilation by a factor of 3: (*Note: Use CAS for the transposition and expanding of* <u>*one*</u> *of these, by-hand algebra for the other two)*

(a) $y = -2x + 5$ (b) $3x + 5y = 15$ (c) $2y = 3(x - 4)$

Verify your answer for <u>one</u> of the above by sketching the original and the image on the same set of axes, showing vertical vector arrows of varying lengths between the graphs. Any point unchanged?

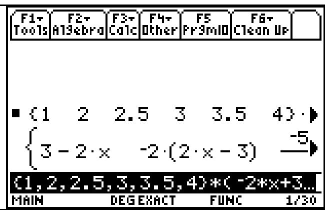
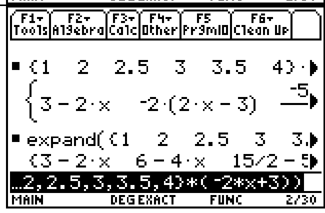
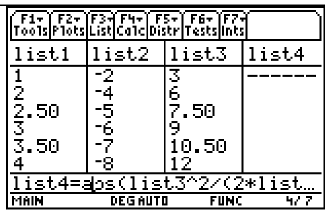
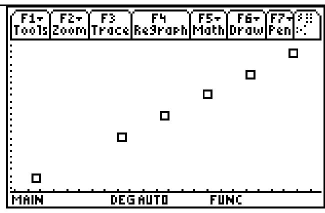
Section 1

2. Consider the function given by $y = ½x + 1$. Complete the table below using CAS and/or by-hand methods:

Dilation factor from *x*-axis	**Equation ($y = mx + c$ form)**	**Sketch graph**	**Value of ◢**
None	$y = ½x + 1$		1
2			
2.5			
3			
3.5			
4			

What pattern do you notice? Do you think it holds for all dilated linear graphs?

3. Now look at the effect of dilating $y=-2x+3$ by the same factors; we use CAS to do all the multiplying, expanding, and analysis in these steps:

Set calculator to exact mode. On the home screen, enclose the dilation factors in set ("curly") brackets, separated by commas, multiplying this group of numbers by the right side of the equation: (some of this is off the screen…)	
When you scroll up to the answer, you scroll to the right to read off the 6 expressions, and note they aren't simplified; so reposition the cursor in front of the previous input and access the 3:expand(command from F2; don't forget to close the curved bracket at the end, and press enter.	
Enter the 6 dilation factors into list1, the 6 gradients (−2, −4, −5, etc) into list2 and the 6 y-intercepts (3, 6, 15/2, etc) into list3. Get CAS to calculate the value of ◢ for these graphs using the formula found in lesson 2, section 2, and to place them in list4 (again, part of the input is off the screen…and note change to auto mode to prepare for regression analysis)	
A scattergram comparing ◢ values (list4 in y) with dilation factors (list1 in x) reveals a linear pattern as shown: Write the equation of this regression equation as found by CAS (you'll need to follow earlier procedures):	

4. If you have time, see if there are similar patterns when you compare gradients (list2) with dilation factors (list1); also, y-intercepts with dilation factors.

<u>Section 2</u>

But does this work for *all* linear graphs? The algebraic capabilities of the CAS home screen can once again be used alongside our by-hand methods to investigate this. Working in teams of 4 (2 using CAS, 2 doing by-hand algebra), and using the letter k to denote the dilation factor from the x-axis, investigate the effect of k on the value of ◢ for any line $y = mx + c$.

Lesson 4

A competition: Finding the line given ◢

Time (minutes)	Teacher activity	Student activity	Materials	Student Learning Outcomes
5	• Recap Lessons 2 and 3 findings: Influences on ◢ by translations and dilations for any general linear function, $m \neq 0$ • Pose questions for discussion (see Worksheet 4)	Discussion within teams	PCs or laptops Poster paper Data projector Worksheet CAS calculator Graph paper Overhead projector and viewscreen adaptor	Leadership and cooperation amongst team members evident as biases for CAS-enabled solutions vs by-hand algebraic solutions are discussed.
60	• Introduce competition questions, monitoring team dynamics and ensuring engagement of all in the process. (See worksheet 4)	Completion of competitive task within team (see Worksheet, Lesson 3)		Will be able to monitor own and team's progress towards a solution to the problem. Will observe need for flexibility and precision within the team.
15	• Monitor team presentations. • Respond to presentations	Present results in teams		Will observe similarities and differences between approaches taken by various teams to solution process.

Mathematics Name ______________________________

Linking Linear Functions and Measurement: An investigation using CAS

Worksheet 4
A competition: Finding the line given ◢

Discussion Questions

1. In your team of four students, discuss the variety of ways the value of ◢ can be altered for any given line, and the effect of that alteration.

2. If you knew in advance what the value of ◢ was, how might you as a team find the equation of the line that produces that area? Will there be only one solution? What methods (including different technologies) might be best used within the team? Should all work together, or should individuals choose their own procedure? How might the results be presented effectively?

Competition

3. Out of the following numbers choose one for your team:

   534   620   56.5   146   22.7   0.4   134

   This is your team's value of ◢.

   In as many ways as possible, find a line or a group of lines which have this particular value of ◢.

   You have one hour to work collaboratively to solve the problem and prepare a presentation to your classmates. The presentation will be judged on the following criteria:

   - Observed cooperation between team members
   - Accuracy of solution
   - Variety of representations of problem and solution (*ie* graphs, technology, algebra, etc)
   - Use of correct terminology and notation in presentation.

Lesson 5

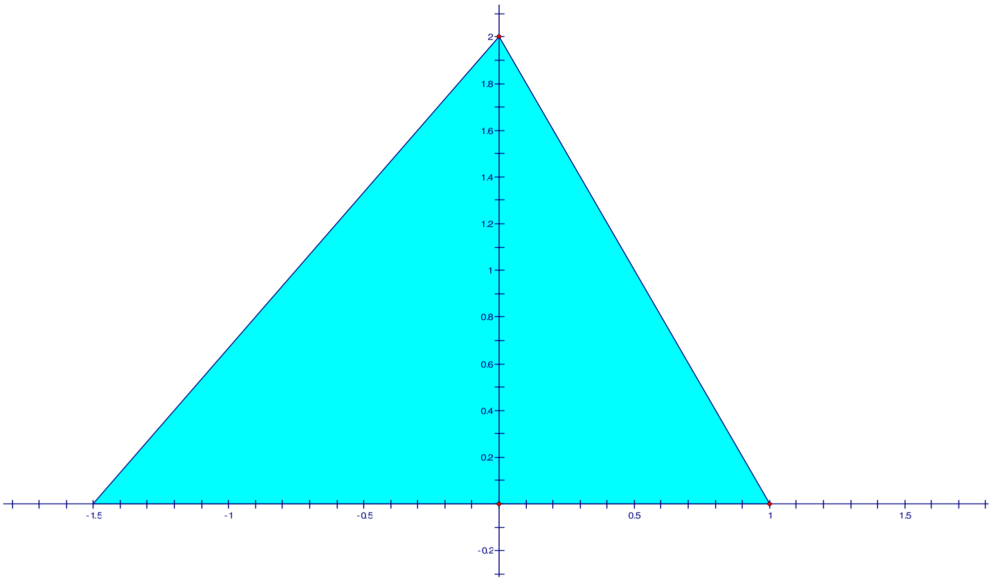
Investigating ▲: Triangle formed by two lines and the *x*-axis

Time (minutes)	Teacher activity	Student activity	Materials	Student Learning Outcomes
10	• Define ▲ as the area enclosed between the *x*-axis and two non-horizontal lines. One of these two lines must have a positive gradient, and the other a negative gradient. (Simple example to illustrate)	Completion of Section 1 (see Worksheet, Lesson 5)	Worksheet CAS calculator Graph paper Overhead projector and viewscreen adaptor	Calculation of ▲ for two lines whose *y*-intercepts are equal.
10	What happens if the *y*-intercepts are not equal? Locating the common height. (Simple example to illustrate)	Finish Section 2 with teacher guidance (see Worksheet, Lesson 5)		Calculation of ▲ for two lines whose *y*-intercepts are not equal.
30	• Introduce problem from Section 3 of worksheet; show use of CAS to formulate the value of ▲ in terms of *m*, the unknown gradient.	Completion of Section 3 with teacher guidance (see Worksheet, Lesson 5)		Will be able to find *m* for any linear function whose *y*-intercept is known, for any other given linear function and a given value of ▲. CAS formulation and solving of equations involving *m* will be privileged.
30	• Introduce fifth investigation task	Work on Section 4 investigation task (see Worksheet, Lesson 5)		Apply knowledge to create a problem situation, similar to that seen in Section 3, where the value of ▲ is known, one line is completely defined, and the other line has a given gradient but an unknown *y*-intercept.

Linking Linear Functions and Measurement: An investigation using CAS

Worksheet 5

Investigating ▲: Triangle formed by two lines and the *x*-axis



<u>Section 1</u>

Consider the triangle formed by two lines and the x-axis as shown above. Find the equations of each of the lines in y=mx+c form; also, find the value of ▲, the area enclosed between the x-axis and two non-horizontal lines. Use

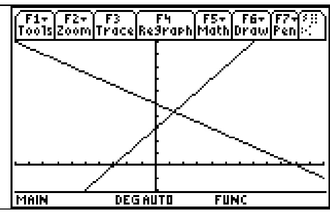
(a) the area formula $A = \dfrac{bh}{2}$;

(b) the individual values of ◢ for each line, using their m and c values;

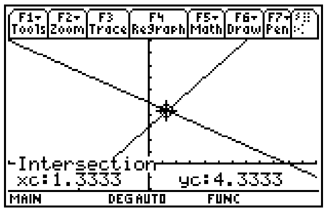
(c) the individual values of ◢ for each line, using the 7:∫f(x)dx command under F5 (Math) on the CAS calculator.

<u>Section 2</u>

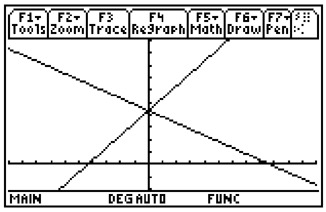
Now let's see what happens when the y-intercepts of the given lines are not equal. Consider the problem of finding ▲ for $y = x + 3$ and $y = -½x + 5$.

The graph to the right is formed by a window where the x-values range from −10 to 12, and the y-values range from −2 to 10. The scales on both axes are 1 unit each.	

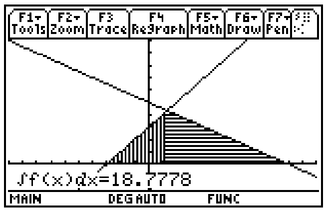
The common height of the two right-angled triangles, which is also the height of the overall triangle, must be found. The *x*-coordinate of the point of intersection looks like it's between 1 and 2; access the Math commands and choose 5:Intersection; you'll be asked to respond to 1^{st} Curve? and 2^{nd} Curve? which, if these are the only two functions you've entered into the y= menu, you just press ENTER to say yes; then, when you're asked for the lower and upper bounds, use the values 1 and 2 for these, or move the cursor accordingly. You should get the screen as shown on the right. Which exact values do these decimals represent?



To find the separate areas, you could translate both lines to the ________ by ________ units so that they would share the same *y*-intercept which would be __________. The new equations could be simplified on the CAS home screen. [Caution here, as exact (improper fraction) values for the translation amount and for the new *y*-intercept will be required for best results]. You'll then get the screen at the right and you can find ▲ using the Section 1 techniques.



You can, however, simply find the 2 values of ◢ using the exact *x*-coordinate of the point of intersection as the upper bound for the first ◢ and as the lower bound for the second ◢. The screen at right shows what you should get once the process is completed on the graph screen of your CAS (note: the first area is found to be 9.3889; and you need to "toggle" to Y_2 before calculating the second area):

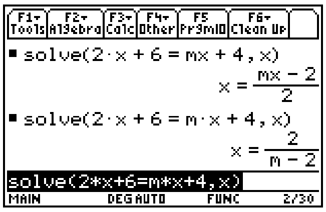
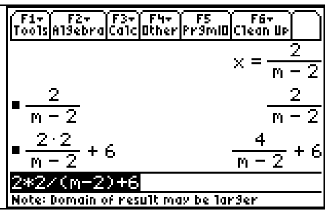


The sum of the individual ◢ values is ____________. What exact value might this be? ______Cutting and pasting of these area values to the home screen for conversion to exact values isn't possible on this CAS; perhaps simple geometry methods for these equations are preferable! Or, you might use the formula for ◢ developed in an earlier lesson. The choice is yours.

Section 3

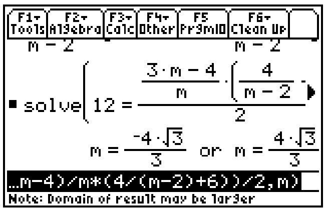
We now consider the task of having a given ▲ value, and trying to find the gradient of one of the two lines.

Suppose we wish to make ▲ = 12 when one of the lines is $y = 2x + 6$ and the other line is $y = mx + 4$.

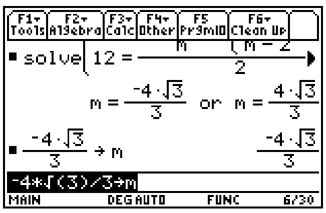
From the work in Section 2, it can be seen that the task requires us to find the point of intersection of the 2 lines. Use the solve(function to set the two "right-hand sides" equal to each other (as they both equal y) and solve for x, in terms of m. [Note, in the first attempt shown here, what happened when a multiplication sign was not inserted between m and x!]	
Substitute this x-coordinate [which was edited leaving off the x = part] into the first equation to then find y, in terms of m:	

The base of the overall triangle is found by subtraction of the x intercepts of the line $y = 2x + 6$ and $y = mx + 4$; these are -3 and $\frac{-4}{m}$, respectively. Since the latter is actually positive (why?) the order of subtraction should be $\frac{-4}{m} - (-3)$, or $\frac{3m-4}{m}$; this too is a positive number. The height of the triangle is the previously found y-coordinate of the point of intersection, $\frac{4}{m-2} + 6$.

Use CAS to substitute these values into the area formula $A = \frac{bh}{2}$, letting $A = 12$ and solving for m:



Which one of these values do we choose? Why are surds involved in the answer?
You can now check this answer by storing the value of m in your CAS thus:



Enter the two functions $y = 2x + 6$ and $y = mx + 4$ into the Y= menu, as they are, and you should see the two lines forming a triangle whose ▲ (confirm with CAS) is 12.

Section 4

This investigation will involve using the CAS to create two lines (one fully defined, and one whose gradient is known to be negative but whose *y*-intercept is unknown), where ▲ is known. The only restriction is that you cannot use the lines used in Section 3, and the value of ▲ must also be a number between 20 and 40.

To help you get started, make these decisions:

My value of ▲ is _____________

My completely defined line equation is ______________________ (remember, its gradient is negative)

My other line has a gradient of ________ so its equation is $y =$ ___x + c. (This gradient must be positive)

A rough sketch of the situation on graph paper, knowing the triangle has to have a specific area, will help you to adjust the window settings of your CAS.

Results:

Lesson 6
The Architect's Problem (Assessment Task)

Time (minutes)	Teacher activity	Student activity	Materials	Student Learning Outcomes
80	Introduce The Architect's Problem Answer questions as required	Begin work on Architect's Problem	Worksheet CAS calculator Graph paper Overhead projector and viewscreen adaptor	Students will apply knowledge gained in previous sessions to solve a practical problem.

Mathematics Name ______________________________

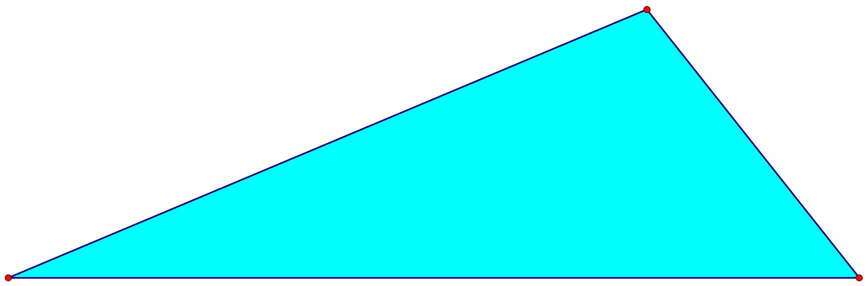
Linking Linear Functions and Measurement: An investigation using CAS

Worksheet 6

The Architect's Problem

A major international corporation is building a new headquarters in Melbourne. They wish to engage the services of a leading local architect to design a large structure that will sit in a prominent place in front of the building.

Required to be shaped as a scalene triangle, with its longest side horizontal, the area of the front face of the structure must be $50m^2$. The angles may not necessarily be identical to the diagram below, but the right side must be the shortest of the 3 sides.



The architect needs to plan her work with care. She is using a mathematical software program to design the sign. Linear graphs represent the sloping sides; while the base is represented by the *x*-axis. Each unit used on the coordinate grid will represent 1 metre.

Your task, as the architect's assistant designer, is to find the equations she will need to enclose an area of $50m^2$. You must be able to demonstrate, with calculations and diagrams, that you have completed the task successfully. This will be done at a board meeting next week with the architect and her client in attendance.

* *

The work will be assessed using similar criteria to those used in the Lesson 4 investigation:

- Observed application to task within class
- Evidence of further work outside class
- Accuracy of solution
- Variety of representations of problem and solution (*ie* graphs, technology, algebra, etc)
- Use of correct terminology and notation in presentation.

Commentary on Curriculum Unit

Linking Linear Functions and Measurement:Investigating using CAS

Introduction

In writing this unit of work I wanted to create an idea that was interesting to teachers as well as to students. Instead of recycling well-worn textbook ideas, I felt it was important to produce a body of material that utilised the diverse range of representations available on a CAS calculator. I have been interested in algebraic representations of geometric ideas for years, and found that I was discovering answers to my own "what if" questions through experimentation. Thus, students throughout the unit are challenged to create, explore, explain and apply the mathematical concepts contained within.

Part of the creative process included the new concepts of ◢ and ▲, which I hope would be given names by the students and teachers who use them. Calculated as specific values early in the unit, they become complex functions of the lines (and their transformations) that form them. The constant "what if…?" posed in the unit serves to remind students and teachers that there are no limits to mathematical thought, and that creativity is now, more than ever before, in the hands of young people. CAS in its "white box" function is an essential part of this process but by-hand methods are recognised as at times being useful and efficient as well.

Geometric, algebraic, numerical and statistical features of the CAS need to be seen as inherently related to each other. Middle secondary teachers who are not mathematics specialists are often likely to keep each syllabus topic in its own little box instead of being part of a much more grand scheme; this often reflects the way they themselves were taught. In the spirit of the VELS "Working Mathematically" domain, the interrelationships of mathematical ideas are front and centre of this unit of work. It is hoped that the step-by-step nature of some of the CAS procedures in this unit can provide a basis for scaffolding mathematical knowledge for teachers as well as for students.

General commentary

Teaching methods in the first five lessons of this unit feature presentations of CAS-specific techniques which will be new to students. As the teacher is expected to use an overhead projector adaptor, she/he can either ask students to keep their screens looking like that which is on the wall, or students can be invited to show/explain their answers. The class thus has an opportunity to diagnose any errors. This again reflects the "teacher as mentor" role and the changing effect on classroom didactics. When students begin their independent or group work in the lesson, the teacher can refer to specific CAS-based skills on the appropriate worksheet which might enable confused students to get back on track.

Team work features strongly in Lesson 4 where the emphasis is on utilisation of the combined skill level of the members. Middle secondary students enjoy competition as long as they feel the contest is fair. They are given a choice in what ◢ value they can use and can nominate the degree to which they use technology to assist them. The teacher needs to provide a variety of resources (as listed) so that the various strengths of team members can be utilised.

Individual independent work is required in the assessment task which is to start in Lesson 6, and might well continue in subsequent lessons. It was felt that by this point, having experienced whole-class teaching and peer group problem-solving strategies, the students will have a full complement of possible strategies for solving the problem. As each student's solution will be unique, the teacher can be more confident that any "out of class" discussions will not provide the solution, but rather a general pathway to that solution.

Lesson-by-lesson commentary

> Note: Each lesson features a worksheet. The teacher will guide the students through the various questions posed and the new CAS techniques described and illustrated with screen dumps. Each ends with an individual or group task which should be completed before the next lesson. Also, it must be stated that given the nature of most schools' timetables, the six lessons allocated to this unit of work might take up to three weeks to present if the class is of mixed abilities. Absences and the usual "I don't have my calculator" mantra make an extended time frame more realistic.

Lesson 1

CAS-assisted graph analysis is presented by the teacher and ◢ is defined as the area of a triangle bounded by a non-horizontal line and the two coordinate axes. The varying value of ◢ is seen to be dependent on the line. A calculus concept (area under a curve) is employed as a quick method of calculating ◢ for CAS-produced graphs. The investigation at the end of the lesson is designed to allow students to explore the symmetry of such triangles produced by parallel lines and their reflections in the axes. As with other lessons, the work is rich with terminology and geometric visualisation of algebraic concepts.

Lesson 2

Revision of the previous lesson's work begins this and all remaining lessons, in an effort to produce familiarity with the techniques as well as the location of the key menu items on this CAS. Horizontal translations are seen to produce a quadratic effect in the value of ◢ which is evident in numeric, symbolic and graphical representations – a critical feature of any CAS. The investigation involves vertical translations without naming them as such, and invites students to use other technologies for their solutions – again, essential in opening up multiple solution pathways.

Lesson 3

The focus of this lesson is dilations. By-hand and/or CAS methods are encouraged as the teacher emphasises the need for a balanced, multi-faceted approach for solutions. Multiple simplifications on the home screen and linear regression are used here, with further pattern exploration invited for more motivated students. A very open-ended investigation enables students to generalise in a manner similar to that done in Lesson 2.

Lesson 4

A competition amongst teams of four students predominates this lesson. Students are challenged to decide roles and approaches which satisfy the goal of producing multiple pathways to a solution. The problem posed is one where ◢ is given but the lines are to be found. This is felt to be a most important activity as reversing the "given/to find" order is an essential skill in higher mathematics. The experience will assist individual students in preparing for the overall assessment task in Lesson 6. The teacher provides a number of different technologies for students use. Assessment criteria as stated give a clear signal to students that they have responsibility to the group. When solutions are given in short presentations at the conclusion of the lesson, the dissemination of various solution strategies will be valuable and creativity will be recognised.

Lesson 5

In a leadup to the overall assessment, ▲ is defined to be the area of the triangle formed by two non-parallel lines and the x-axis. Translations are used to see ▲ as the sum of two separate values of ◢. The limitations of some CAS techniques are pointed out here and in other lessons, and students are challenged to make an informed choice based on all knowledge they possess. Discernment between two distinct CAS-enabled solutions is required, another essential skill for today's students. The investigation features student choice of a ▲ value and a subsequent search for two lines, one of which is completely defined and the other where the gradient is unknown. CAS-enabled solutions of difficult equations are seen throughout the lesson.

Lesson 6

The main assessment task dominates this lesson. Students take on a real-world problem involving area and apply the skills and techniques learned previously to arrive at a solution which they must justify in multiple ways. The link between theory and real-world measurement is emphasised so that true meaning can be derived from the work.

Conclusion

The use of CAS at most secondary schools has filtered down to Years 9 and 10. Golden opportunities exist to make the algebra come to life for and mean something to more students. As senior mathematics examiners are re-evaluating exam questions in light of what CAS can do, middle secondary teachers also need to question the isolated manner in which topics are often taught. Integrating various branches of mathematics into problem-solving can give new and deeper understanding to our students. CAS use enables these younger students to access rich mathematical activities. Hopefully this body of work will make a contribution to that effort.

Roger Wander